Early Area Measurement Instruments

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1. Area measurement in the 19th century

Already more than 200 years ago civil engineers had to determine areas of properties, on maps or any other type of figures. In 1853, Bauernfeind [Bau1853-1] estimated the demand for these types of calculations at about 1 trillion measurements of figures per year in Europe. Even if this number might be too high[1], it gives an indication of the urgent need for devices enabling fast, efficient and accurate methods to determine areas of figures.

During the first half of the 19th century, the first integrating planimeters were invented. There are many different types of planimeters of this category, e.g. polar planimeter, linear planimeter, disc roll planimeter, and many more. These planimeter types have one common characteristic. The area of a figure is determined by tracing the (closed) boundary of the figure, while the measuring part of the planimeter continuously integrates the enclosed area of the figure. After reaching the starting point on the (closed) boundary the measurement device shows the value of the surrounded area (up to a constant to be multiplied with the result). A more detailed description of the usage of a polar planimeter can be found in [HerZer2002].

By 1814, the Bavarian surveying engineer J. M. Hermann had invented such an integrating planimeter, but this invention had been forgotten. The corresponding description was found again only in 1855. Meanwhile, in the year 1827, Oppikofer made the same invention again. In 1824, Professor Titus Gonella independently invented yet again a similar mechanism for integrating planimeters.

The dynamic of the development and improvements on the (integrating) planimeter has been well described by Durant-Richard in his article [Dur2010]: *"This idea, first proposed by Gonnella in 1825[2], was reinvented in 1849-50 by the Swiss engineer Kaspar Wetli (1822-1889) [...]. Successively modified for precision and usefulness by Simon Stampfer (1790-1864), and by the astronomer Peter Andreas Hansen (1795-1874) in Seeberg/Gotha, the device was manufactured and commercialized with immediate success in the 1850s by the polytechnic workshop of Georg Christoph Starke (1794-1865) in Vienna, and later by Hermann Ausfeld [...]. Approximatively five hundred items were in circulation at that time."*

But this overview is by far not complete. From 1833 onwards Oppikofer partnered with the mechanic Ernst optimized the Oppikofer planimeter. In 1854 Amsler-Laffon invented his polar planimeter, which became a worldwide success, while one year later the polar coordinate planimeter invented by Dechler [Dec1855] had not been successful [Ams1856-2].

Even though these types of devices are easy to use and operate with accuracy, the spread of the usage of these devices took some time. One reason might have been that the mathematical theory behind the usage of the integrating planimeter is more complex and some potential users might have doubted the correctness of the results for measured areas (see [Tru1865], pages 187-188). Another reason might have been the high price (e.g. 400 Francs in 1833 for the Oppikofer-Ernst planimeter).

Therefore, the 19th century can be considered as the century of planimeter inventions. The time line (Figure 1) provides an (incomplete) overview of planimeter inventions during this time period. But again, the representation should not lead to the wrong impression that the invention of planimeters had happened in a sequence. Inventors designed their devices independently and in parallel. Especially there have been two parallel main streams of development:

- Integrating planimeters (blue marked inventions in Figure 1)
- Non-integrating planimeters (orange marked inventions in Figure 1)

[1] The source of the estimation cannot be validated.
[2] This was already an improvement of Gonella's original invention by 1824.

Due to the parallel and non-sequential inventions, the remainder of this article will not chronologically follow the history, but try to explain the methodologies and mechanical approaches from a user's perspective. For the chronological order the reader is asked to consider Figure 1 while reading the paper.

Additionally it is worth mentioning, that in the remaining part of this article, we will not look at integrating planimeters, but only describe much simpler methods and devices used to determine the area of closed figures.

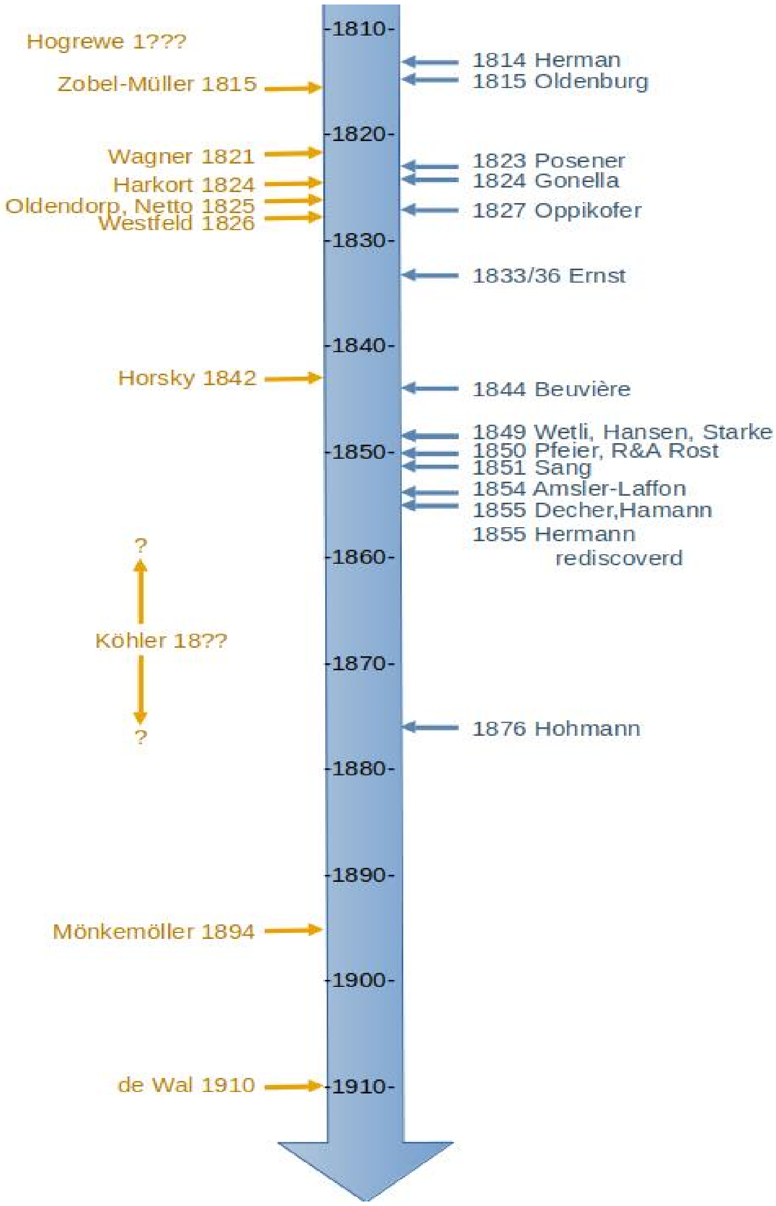


Figure 1: Time line of early planimeter inventions (orange non-integrating planimeters; blue: integrating planimeters).

2. Area measurements by estimation or counting

One easy, extremely cumbersome and error prone approach is the usage of "estimation squares". This method uses transparent material (e.g. a thin horn plate or glass) with fine squares. According to Hunaeus [Hun1848, Hun1862, Hun1882] the initial idea for this approach had been introduced by Hogrewe. Putting this transparent square paper (or glass) over the figure to be measured, one has to count the number of millimetre squares lying inside the figure under investigation. The number of squares multiplied by the square width and by the scale of the figure gives the real result for the area.

If you would like to experience this challenge, simply take a sheet of millimetre graph paper, draw an arbitrary closed area on the sheet and then start counting the squares inside the closed area. Certainly after a short time you won't enjoy the task. You will face counting errors and on the boundary of the figure for each and every square which is only partly inside the area you will need to decide whether to count the square or not. Of course, this is a significant source of errors. Therefore, the described method is not considered as a measurement but an area estimation approach.

3. Area measurements by approximation

One obvious improvement for area measurements is the following. Decompose the area into a set of simple figures with known or easy to calculate areas. Therefore, in the 19th century, efforts were made to invent devices for determining areas of arbitrary, but plane figures. A good overview of available solutions in the first half of the 19th century can be found in [Fis1868].

Especially figures with straight bounding lines can be approximated by rectangles and triangles. Of course, a demand for increasing accuracy and irregular boundaries dramatically increase the number of performed measurements and corresponding calculations. The huge number of these calculations led to a high risk for error prone results again, ending up in a similar situation as with square millimetre papers.

A first simple approximation is the application of Simpson's formula:

$$\int_{x'}^{x''} f\,(x)dx = \frac{h}{2}[y_0 + y_{2n} + 4(y_1 + y_3 + y_5 + \ldots + y_{2n-1}) + 2(y_2 + y_4 + y_6 + \ldots + y_{2n-2})]$$

This formula sums up equidistant strips (of width *h* under a given curve *f(x)* in the interval *[x',x'']* (see Figure 2). Obviously, for the calculations 2n measurements[3] for the values y_i and lengthy additions are required. Increasing the accuracy could be achieved by using smaller strips, e.g. using strips of width *h' = h/2* doubles the required measurements for y_i and doubles the number of additions to be done.

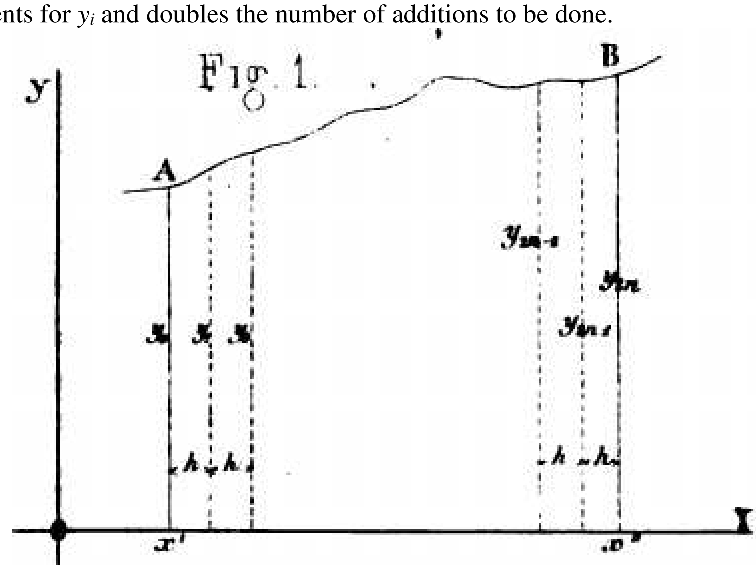


Figure 2: Usage of Simpson's Formula.

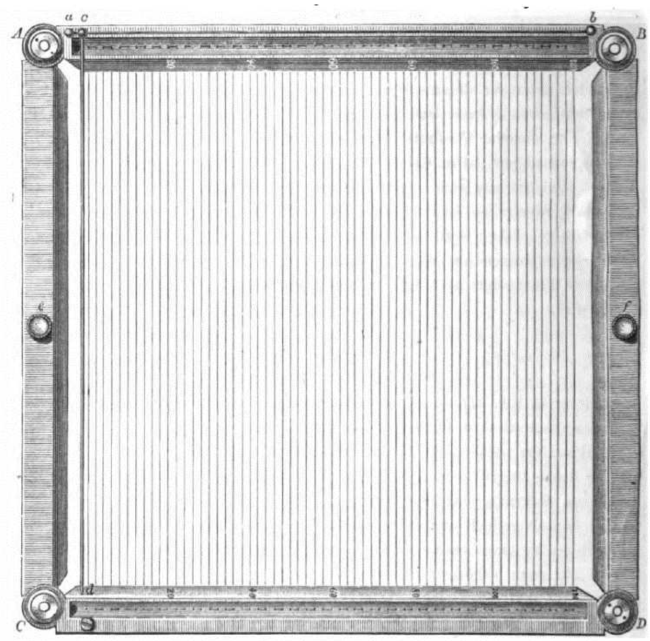
Another, more complex approach is the decomposition of the area into triangles and rectangles. Again measuring the size of these figures, separate area calculations plus adding all the results, not missing any sub-areas, is a difficult task. Therefore, inventions based on Simpson's approach focused on mechanical addition of the length of the strips.

[3] To be correct: The number of measurements is 2n+1.

4. Harp Planimeters also known as Thread or Hair Planimeters

In 1825 Eduard Oldendorp (*17.10.1795; †1881[4]) introduced a so-called Harp Planimeter (or Thread or Hair Planimeter) which is a device directly supporting the above described approach. The device consists of a frame with many equidistant, vertically strung threads. Originally, Oldendorp used silk threads [Hun1862]. For performing a measurement of an area, the operator puts the device over the area to be measured. The threads divide the area in strips of identical width (the distance between two threads). For measuring the length of the strips and adding all lengths, a special device called a "Hunderterzirkel" (Figure 4) or Planimeter Compass ("Planimeterzirkel") is used. The article [Sch1832] describes in detail the application of Oldendorp's planimeter together with the planimeter compass. The compass can open up to a fixed position, which corresponds to a maximum length to be measured. The maximum opening angle of the compass is set by a corresponding screw (see e1 in Figure 4). The operator uses the compass to add the length of each stripe while opening the compass more and more. Only when the maximum compass opening is reached, the gear (see g in Figure 4) turns one tooth forward. A special spring (see h in Figure 4) controls the stepwise movement of the gear. So the gear (with 50 teeth) can be considered as a kind of counter for complete openings of the compass during an ongoing measurement. Each time the compass has been opened completely, the operator closes the compass and continued the measurement of the recent strip by reopening the compass up to the length of the strip. The result for the measured area equals:

("No. of complete compass openings x length of complete compass opening" +
"length of last opening of the compass") x "distance between two threads of the harp planimeter"



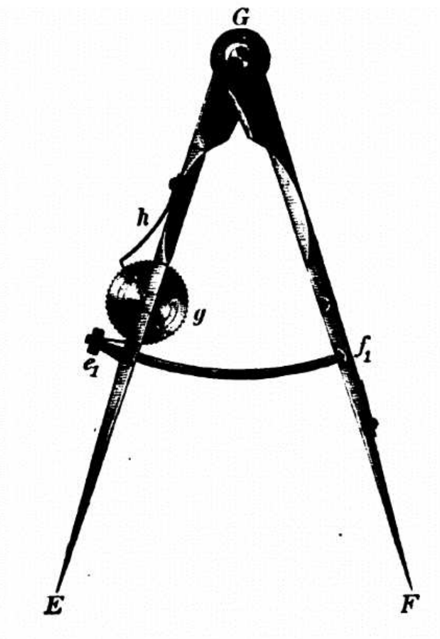


Figure 3: Oldendorp's Harp Planimeter (Source: [Hun1862]). Figure 4: Oldendorp's Planimeter Compass (Source: [Hun1862]).

Of course, instead of Oldendorp's planimeter compass, any kind of compass of appropriate size without the gear but with adjustable maximum opening angle could be used. In this case the operator needs to count the number of complete compass openings. Besides Eduard Oldendorp the Austrian cadastral officer Alder introduced a similar solution at about the same time. Lemock ([Lem1849] p. 66-69) describes Alder's construction and the usage in detail. Especially, he mentioned Alder used black and red coloured horsehairs as threads, leading to a better tracking of strips to be measured. So the risk of missing strips or measuring twice was reduced.

[4] The year of death is not confirmed.

Figure 5: Harp planimeter possibly by Neuhöfer & Son, Wien. The original threads (horizontal line of thread holes are visible on the top and bottom) had been exchanged by a transparent plastic foil (Source: Collection Zerfowski).

During the same period of time even simpler solutions for solving area measurements have been proposed, e.g. the Austrian cadastral[5] surveyor Johann Paul Posener (* Margonin, Prov. Posen (Poland), 21. 2. 1793; † Graz (Austria), 20. 3. 1861)[6] and Prussian Lieutenant Netto described solutions which only required a scale and a triangle (see [Poe1823], [Lem1849] p. 61 and are easier to understand [DreHae2009]). The triangle is stepwise shifted over the area, along the fixed scale. After each shift the length of the section of one triangle side inside the figure is measured and added (e.g. with a planimeter compass).

There are several variants of Harp Planimeters working in the same manner, but with small modifications. So instead of threads, transparent paper with printed equidistant lines (see Figure 5) or glass panes with imprinted or engraved lines have been used.

The use of planimeter compasses often result in damaged threads. Therefore, original thread planimeters were modified by their users. Exchanging a transparent paper (damaged by the compass) was much easier than exchanging damaged threads or horsehairs.

[5] Cadastre is a technical term for a set of records showing the extent, value and ownership (or other basis for use or occupancy) of land. A modern cadastre normally consists of a series of large-scale maps or plans, and corresponding registers.
[6] See: https://www.biographien.ac.at/oebl/oebl_P/Posener_Johann-Paul_1793_1861.xml

5. Harkort's Universal Planimeter

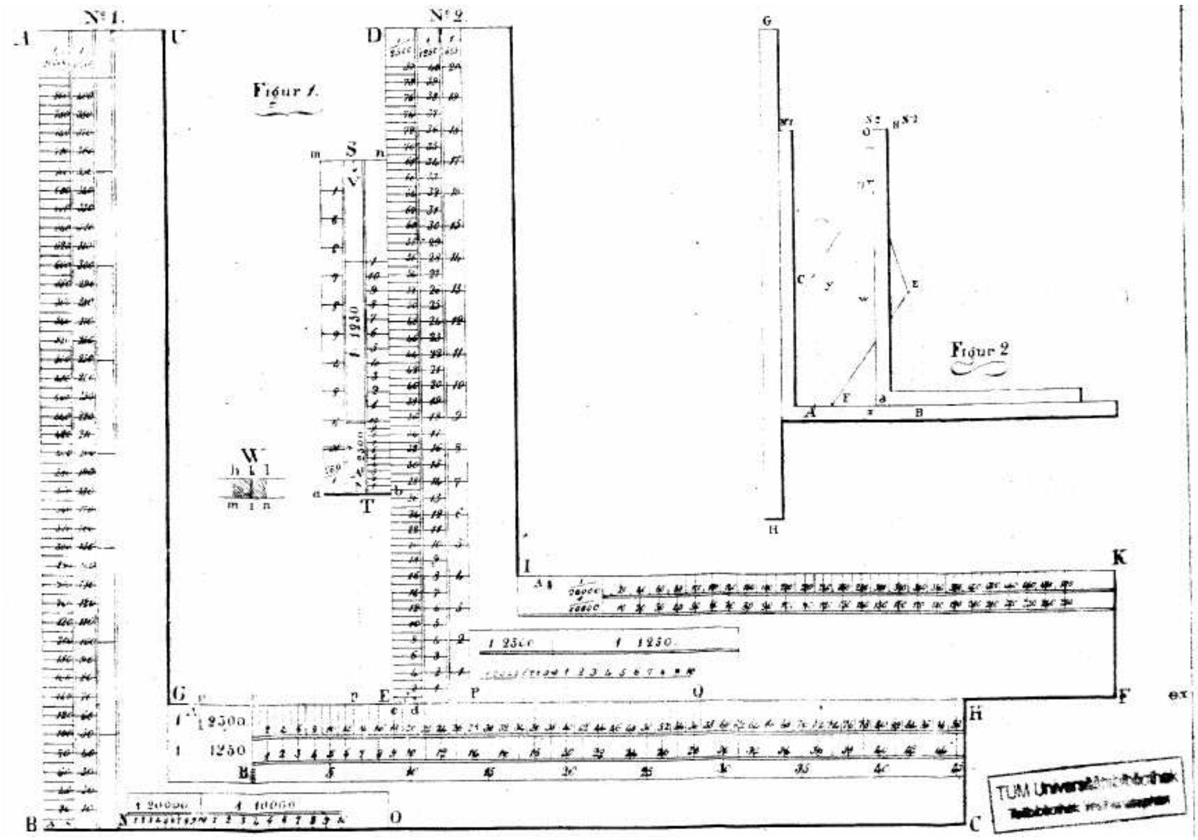


Figure 6: Drawing of Harkort's Universal Planimeter (Source: [Har1824]).

In the booklet "Das Universal-Planimeter" [Har1824], Eduard Harkort[7] describes a device which substitutes reduction circles, other planimeters and especially glass planimeters. The selling argument for the Universal-Planimeter was its wide range of applications. It has been used for enlarging, reducing or copying maps, and determining areas. One important advantage was the fact, that during these operations the original map has not been damaged, e.g. by needles of reduction circles or other devices. At that time original maps had been of high value and any damage should have been avoided. Even today original maps from the 19th century are still valid governmental documents, e.g. in the state of Baden-Württemberg, Germany.

How does Harkort's planimeter work? Figure 6 gives an impression of the appearance of the device. It consists of two equally sized, L-shaped, wooden right-angles (ABC and DEF). Between the two right-angles, an additional slide (S) is located. Neither the angles nor the slide are connected to each other, but are placed close to each other and are sliding along their edges, e.g. the edge EF of the second right-angle moves along the edge GH of the first right-angle and the slide S is moved along the edge DE of the second right-angle. All three parts are made of wood, carrying different scales made from ivory or glass with underlying paper.

For determining the area of an arbitrary quadrangle (CDEF) an additional ruler GH is used (see upper right image in Figure 6). By sliding the components along their edges, the operator measures distances within the quadrangle by using the scales. With these measurements, the area of the quadrangle is manually calculated. In case the original area is not a quadrangle, the operator first need to decompose to quadrangles. Afterwards for each quadrangle the area is determined and summed up.

Over several decades, many publications referred to Harkort's Universal-Planimeter. Unfortunately, the author of this article is not aware of any existing instrument but would be glad to receive any information on this.

[7] For details on Eduard Harkort see [Rud2009].

6. Wagner's Planimeter (1821)

In 1821, Wagner published the description of a completely different planimeter device [Wag1821]. Instead of equidistant strips, Wagner went for the approach to split the area of interest into triangles and then determined the areas of the triangles and trapezoids using his device. The mathematical foundation is geometrical proportions, especially proportional triangles.

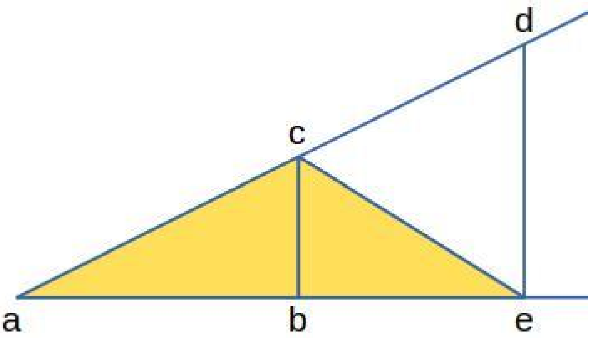


Figure 7: Proportional triangles and calculation of area of the orange triangle

According to Figure 7, the area of the orange triangle A_{ace} can be calculated by

$$A_{ace} = \frac{\bar{ae} * \bar{de}}{2} - A_{cde} = \frac{\bar{ae} * \bar{de}}{2} - \frac{\bar{be} - \bar{de}}{2} = \frac{\left(\bar{ae} - \bar{be}\right) * de}{2} = \frac{\bar{ab} * \bar{de}}{2}$$

Consequently, for any fixed position of b the area of the orange triangle only depends on the position of the vertical line de.

Figure 8 shows the original drawing of Wagner's planimeter. The device consists of two orthogonal metal frames on which two triangles JKL and FGH can be moved. The first can move vertically, the second horizontally. A metal scale MN is fixed at the point C and can rotate around the fixation point C. The fixation point C can be moved horizontally on the edge of the triangle JKL.

To measure the orange triangle in Figure 7, the operator puts Wagner's device on the triangle, such that the point "a" is located at the origin of the metal frame. Afterwards the triangle JKL is moved vertically such that the lower horizontal edge is touching the point "c". Now the right metal triangle is horizontally moved such that the vertical scale touches the point "e". Finally the fixation point C is horizontally moved on point "c" and the scale MN is turned such that it goes through the origin of the metal frame. Therefore, this diagonal scale lays on line "ad" in Figure 7.This intersects the scale MN with the vertical scale (point "d" in Figure 7). At the intersection "d" the area of the triangle ace can be read on the vertical scale.

Wagner mentioned that using his device a single person can calculate 600 to 800 trapezoids per day. Without his device, this would require two to three working days.

Already in 1823, some criticism of the device had been published in official documents. During revisions of performed measurements and calculations with Wagner's planimeter, errors had been observed. While errors in opposite directions eliminated each other, these single errors still stayed with dedicated land property calculations (see [Koh1858]). For this reason, by 1821 governmental orders had been given, to recalculate a significant number of land properties to eliminate these errors. Obviously, this has not been the best advertisement for Wagner's device.

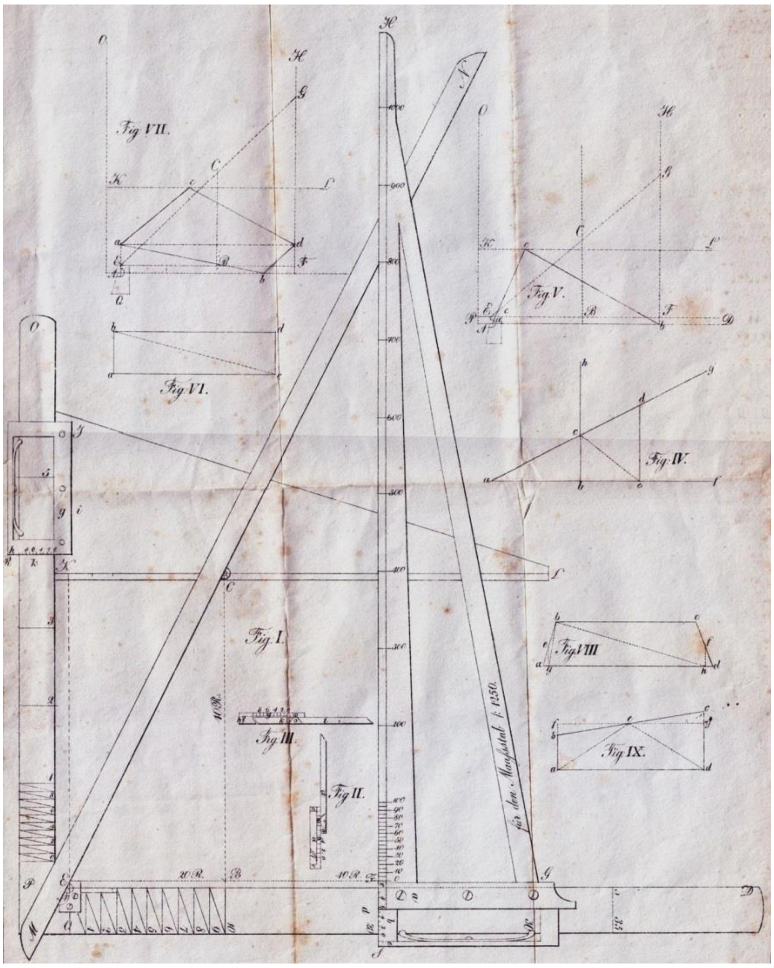


Figure 8: Drawing of Wagner's Planimeter (Source: [Wag1821], scan provided by Andries de Man)

7. Netto's Approach (1825)

Four years after Wagner's publication, Friedrich August Wilhelm Netto introduced a much simpler method in his book [Net1825]. InFigure 9 no. 13 the proposed method is shown. A 15 to 18 inch long metal scale (AB) and a metal triangle (ACD) of the same thickness are required. The ratio of the length of the cathete[8] (AC) to the hypotenuse (AD) equals (AC) / (AD) = 1 / 5. In the centre of the hypotenuse an index marker (F) is given. Shifting the triangle along the metal scale such that the index (F) with each shift moves to the next inch mark on the metal scale (AB). Consequently the triangle's cathete (CD) divides the area into equidistant, parallel stripes. In real applications not the entire strips, but the intersection with the area's boundary is marked. The distances between the two intersection points of each strip is manually measured and added according to Simpson's formula.

Of course, this method is quite cumbersome and comes with a high risk for errors. The equipment (it should not even be called a device) is extremely cheap. But in today's business language, the "Total Cost of Ownership" is definitely high, due the high manual effort and errors in the measurement.

[8] Google translated from the German: 'kathete', one or other of the two legs of a right angled triangle that is not the hypotenuse. According to Wikipedia: The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).

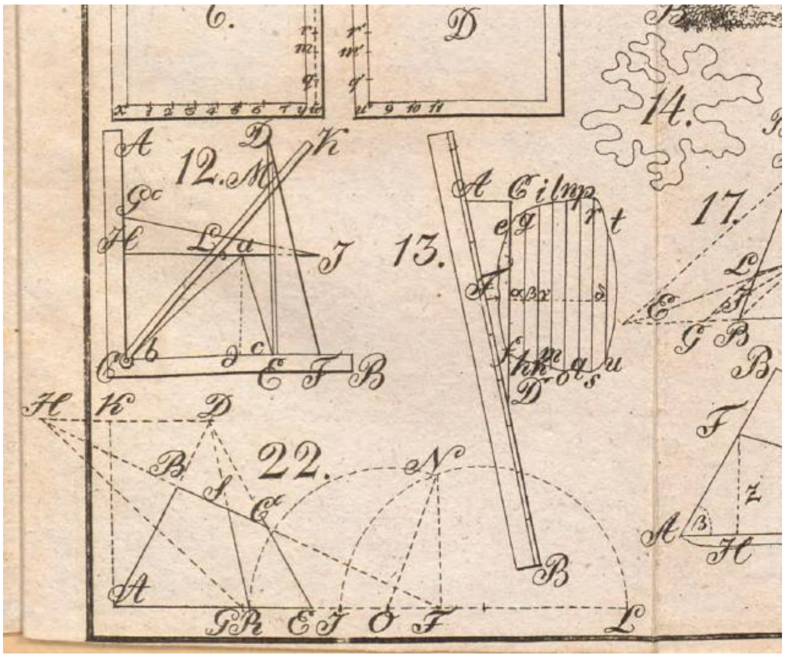


Figure 9: Netto's planimeter approach (figure no. 13) and a complex area (figure no 14). Figure no. 12 shows Wagner's planimeter. (Source: [Net1825]).

Netto also mentioned that his approach might not be suitable for arbitrary complex shaped areas. He gave an example of such an area (Figure 9 upper right area numbered 14.) and explained an alternative method to determine the area.

1. Copy the boundary of the area on a sheet of tinfoil
2. With a sharp tool precisely cut out the copied area
3. Draw a square of known size on the same sheet of tinfoil
4. Cut out this area as well
5. Weigh both cut tinfoils on a high precision chemical gauge.

If "a" is the known area of the tinfoil square, "s" is its weight and "f" is the weight of the tinfoil area to be measured, than the searched area of the figure is calculated by

$$x = \frac{a * f}{s}$$

Here it is worth mentioning, that tinfoil has to be used. The usage of normal paper would not have been possible due to the lack of homogeneity in the material even within one sheet of paper.

8. Planimeter by Horsky (1842)

In 1842 the Austrian cadastral officer Franz Horsky applied for a patent of his planimeter [Wag1842]. Horsky was not satisfied and not convinced by strip measurements. He started his article [Hor1850] with the statement (translated from the German):

"Many years of experiences in the service of the royal land registry office have taught me that the mutiplication in the calculation of recorded plots is time consuming and has disadvantages for the business. This is because of calculations of equidistant strips, which anyhow can only be applied for a few plots and for most plots could not be used at all."

Based on this motivation Horsky constructed a device shown in Figure 10. The device required that the area of interest had to be divided into quadrangles. With Horsky's planimeter these quadrangles are measured without any required multiplication.

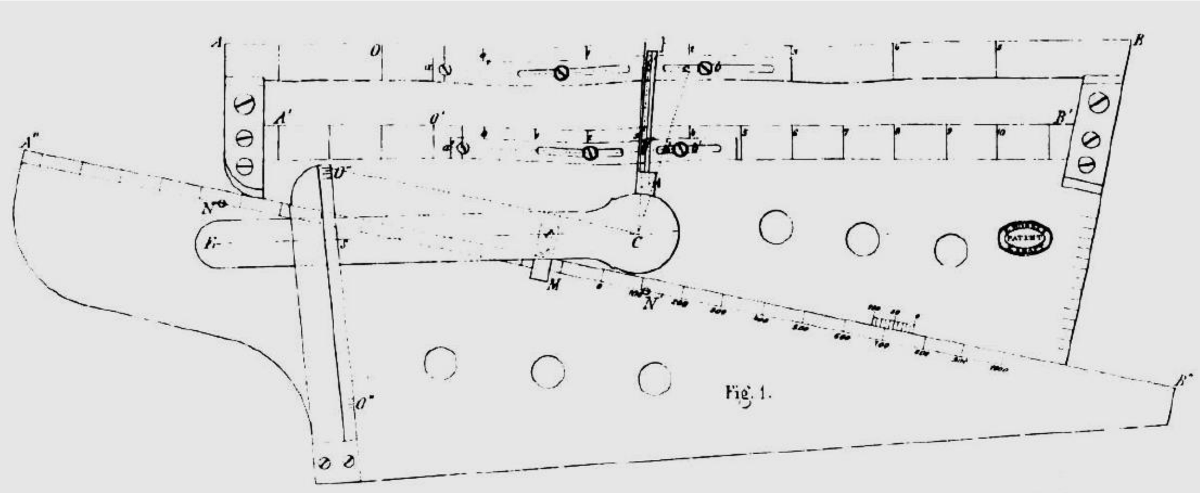


Figure 10: Horsky's planimeter (Source: [Hor1850]).

Within three month after Horsky's publication [Hor1850], a report based on practical experiences was published in the same journal. Professor Stampfer and Oberst Hawliczek [HawSta1850] provided an indifferent response. Stampfer and Hawliczek mentioned that the application of Horsky's planimeter is difficult to operate, and the accuracy of the device is considered as sufficient. A significant efficiency improvement compared to the usage of Posener's device has not been confirmed. Therefore, the reviewers did not give a strong recommendation for Horsky's device, but stated that the instrument might be used.

9. Ringmesser by Westfeld (1826)

In 1826, O. Westfeld introduced a different device, the so-called "Ringmesser" (Engl:. ring measuring device). While Oldendorp measured the area by dividing the figure into equidistant parallel strips, Westfeld divided the area into equidistant, concentric ring segments and measured and added the length of these ring segments

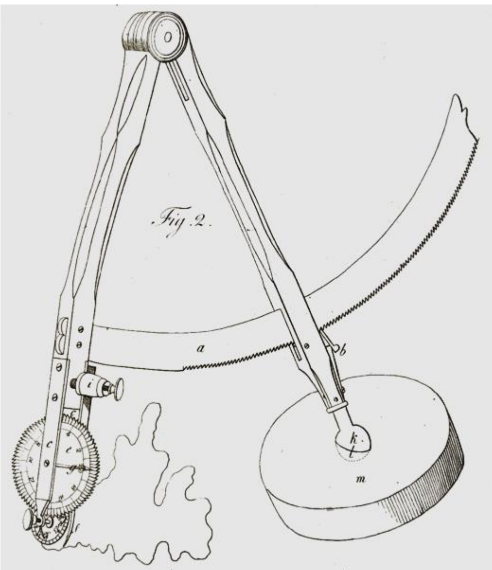


Figure 11 Westfeld's Ringmesser. (Source: [Wes1826]).

In his own publication [Wes1826], Westfeld describes the compass like device and its usage. During a measurement one part of the Ringmesser stays fixed close to the figure of interest (see right part "m" in Fig 11.)[9]. On the other end of the compass like device a measuring wheel (small wheel in Fig 11.) is mounted. Above the small wheel a bigger wheel with 80 teeth is mounted, which counts the number of complete revolutions of the smaller measuring wheel.

The operator starts the measurement opening the device up to the out most part of the figure. Now the device is used similar to a normal compass and the operator runs the small wheel inside the figure to be measured. As a result, the length of the ring segment is measured on both wheels. For the next ring segment measurement, the device is carefully closed a little. To ensure an equidistant closing of the device, the metal guiding segment of the device (part "a") carries 80 to 100 teeth. The operator closes the device until a spring (part "b") snaps into the place of the next tooth. Now the next ring segment inside the area is measured.

[9] Don't get confused by the strange, wrong perspective of the drawing from [Wes1826].

This process continues until the last ring segment inside the area is measured. The sum of the length of all ring segments are shown by the counter wheel and measurement wheel. Depending on the figure’s scale, the result of the measurement might need to be finally multiplied by a constant factor.

Westfeld structured his publication in three sections. The first section describes the mathematical foundation of the Ringmesser. In the second section Westfeld describes the device only verbally (without figures). This is done intentionally to avoid other mechanics easily copying the device. Instead Westfeld looked for business and offered the device for 8 to 10 Konventionsthaler[10]. According to the official conversion table for purchasing power equivalent for German currencies [Bun2021] the amount of 1 Thaler corresponds to 42.4 Euro (in 2020). Konventionsthaler in 1825 correspond to approximately 424 Euro in 2020.

In the third section Westfeld describes the usage and the accuracy of the device, which is given as ⅓ % (Hun1862]). In the same book [Hun1862] Hunäus also mentioned that Westfeld’s Ringmesser did not widely spread in the market. Trunck indicated ([Tru1865] p. 184) that the exact positioning of the measurement wheel on the drawing was difficult. Additionally Trunk complained that, depending on the opening angle of the Ringmesser, the measuring wheel runs on a peripheral edge of the wheel, resulting in different length for one revolution and therefore leading to inaccurate results.

With respect to Westfeld’s approach it should be noted, that nearly 100 years later, in the year 1925, another planimeter based on measured ring segments (called a Vector planimeter) had been invited by Schnöckel [Sch1925p].

10. Glass Planimeter by Mönkemöller (1894)

Even in the late 19^{th} century extended versions of Oldendorp’s Harp planimeter have been realised. One example of these devices is the Glass planimeter by Mönkemöller, who filed the corresponding German patent 78714 [Mon1894p] on 8^{th} April, 1894. The planimeter came with a two page manual [Mon ????] by the manufacturer Max Wolz at Bonn, Germany, a workshop for precision mechanics at that time. Mönkemöller also tried to convince potential customers (especially surveying engineers) by describing the advantages of the device in the German Zeitschrift für Vermessungswesen (Engl.: Journal of Surveying) [Mon1895].

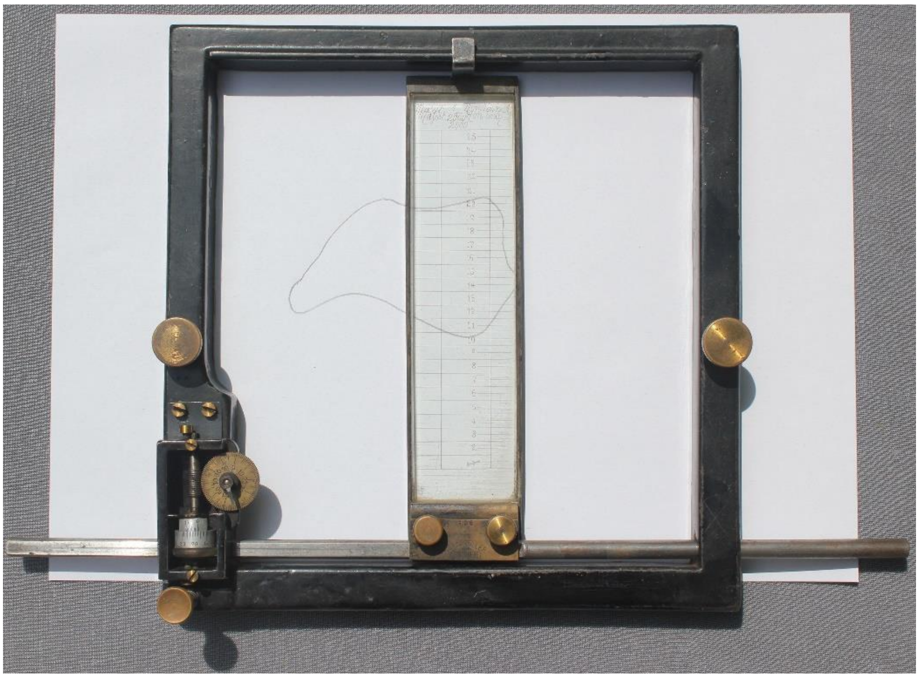


Figure 12: Mönkemöller's Glass Planimeter (Source: Collection Zerfowski).

[10] Konventionstaler had been a widely used currency, defined by Austria & Bavaria and spreading to Saxony as well. From 1838 onwards in Germany Konventionsthaler have not been minted any more.

At this time, integrating planimeters had already been in place. Therefore, it seems a bit strange that even at this late period of time (see Figure 1) this non-integrating planimeter entered the market.

The planimeter (also described in [Hus1896]) consists of a frame with a guide rail at the top and a movable metal bar at the bottom. Between the rail and the metal bar, a metal framed glass plate is vertically mounted. The glass plate carries numbered, equidistant horizontal lines and two vertical lines. The glass plate is fixed on the moveable metal bar (see two brass screws at the bottom of the glass planimeter in Figure 12). Therefore, while moving the glass plate to the left or right the metal bar moves accordingly. Pressing a knob (see Figure 13) while moving the glass bar the measuring wheel rolls on the moveable metal bar and adds the horizontally moved distance. The planimeter comes with two exchangeable glass plates for different scales (see Figure 14).

For performing a measurement, the device is placed above the area to be measured (see Figure 12). Similar to Oldendorp's principle, the horizontal lines on the glass bar divides the area into equidistant strips. Moving the glass bar such that the vertical line in the first horizontal strip intersects the right boundary of the area. Now pressing the knob (see Figure 13) and while holding the knob, the glass bar together with the fixed metal bar are moved to the left until the vertical line intersects the left boundary within the first strip. Reaching this boundary the knob is released. The measuring wheel added the length of the 1st strip within the measured area. Afterwards the same process is done for the second strip until the last strip covering the area under investigation. Finally, the measurement wheel and the attached Vernier have added the sum of all strip lengths of the analysed area. Multiplying this result with the constant width of the strips gives the measured area of the analyzed figure.

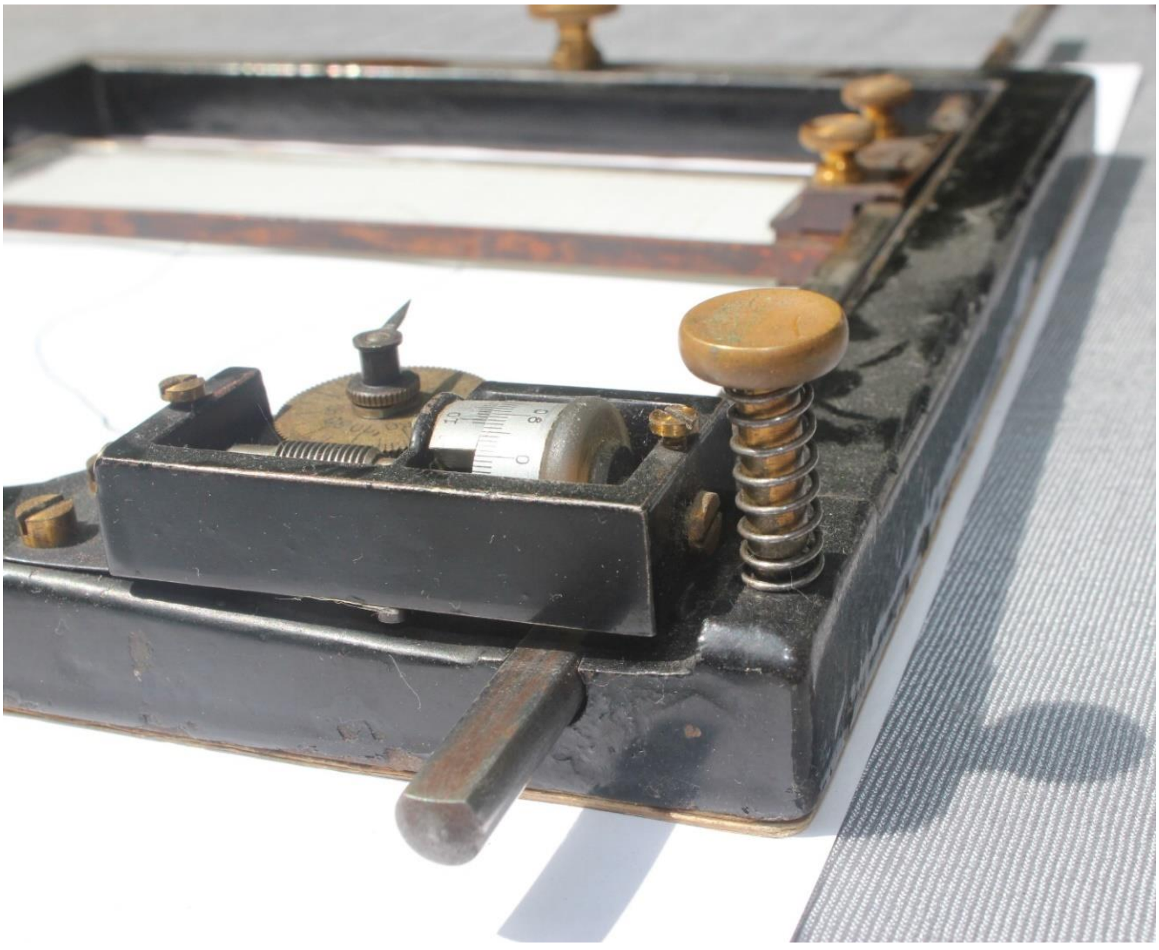


Figure13: Mönkemöller's measuring wheel: Pressing the knob puts the wheel down on the movable metal bar (Source: Collection Zerfowski).

In an article from 1899 [Ham1899] Hamann reviewed the performance of the device. According to his experiments he experienced a measurement error of about 0.1% of the total area, which was similar to the accuracy of integrating precision planimeters available at that time. Hamann mentioned as the main source of error, that one might easily miss a strip or by chance measure a strip twice.

Even though Mönkemöller's planimeter received a good report by Hüser [Hus1896], mentioning the high accuracy and efficiency, the device seems not to have spread widely which was already mentioned by de Wal in 1910 [Wal1910] and Hammer [Ham1911]. In his own article, de Wal proposed another approach using a slide rule instead of a planimeter compass. But de Wal's idea left even fewer traces in the history of planimeters compared to Mönkemöller's device.

Figure 12 and 13 show an original Mönkemöller glass planimeter from the author's collection. So far, the author is only aware of four additional existing examples:

- at the collection of the University of Hannover, Germany [Hee2003],
- at the "Rechnerlexikon" (see: http://rechnerlexikon.de/en/artikel/M%F6nkem%F6ller_Planimeter),
- at an auction at the auction house Mehlis (28th May 2018) , see: https://veryimportantlot.com/en/lot/view/zwei-geodatische-messinstrumente-323588), and
- a much younger device sold for 506 Euro on eBay (2nd August 2021) (Figure 14).

Almost certainly, these are not the only existing examples. The author would be very glad to receive additional information on further existing devices.

During a personal communication with Norbert Kreitel some more information on the manufacturer Max Wolz has been collected. The company, "Herbert Kreitel Bonn" took over parts of the original Max Wolz Company. The founder Herbert Kreitel, (father of Norbert Kreitel) had been operations manager at Max Wolz from 1958 until the end of 1962. From January 1963, Mr. Kreitel continued parts of the business under the name "Max Wolz - Feinmechanische Werkstätten - Owner Herbert Kreitel" (see [Kre2021]). The manufacturing of the glass planimeter had already been stopped a long time before the hand-over of the company.

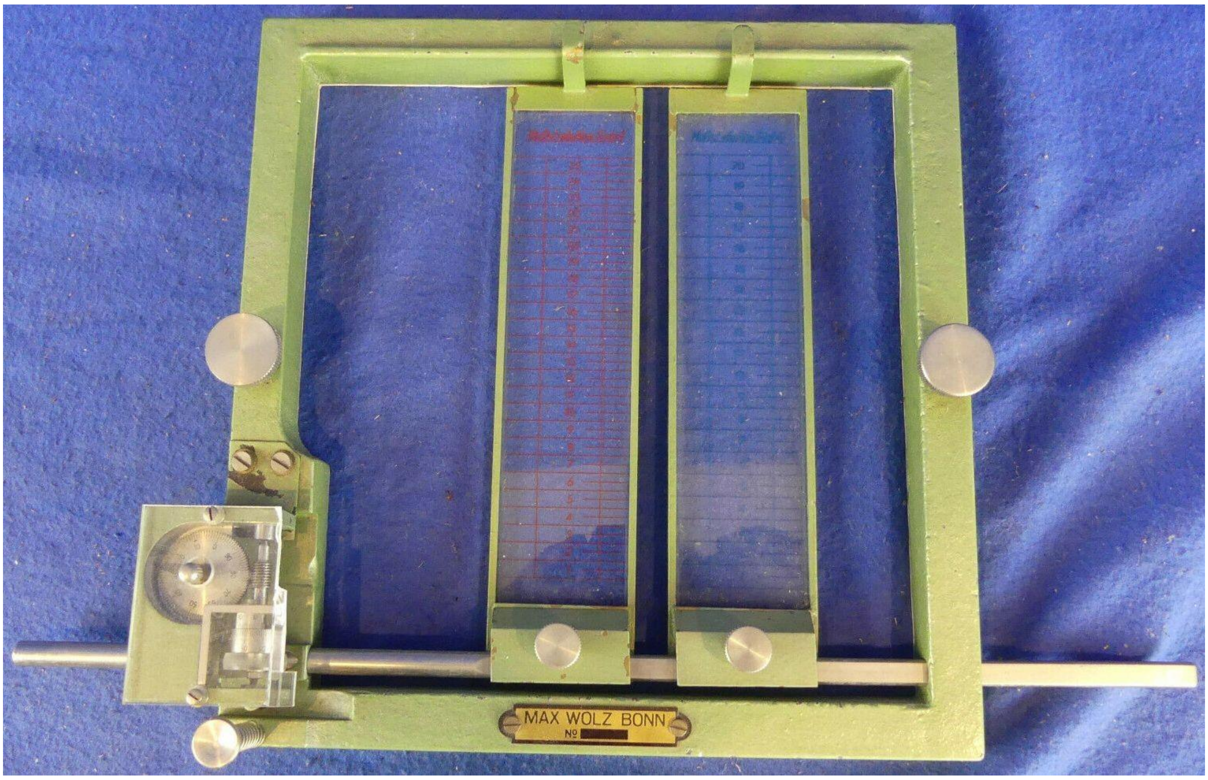


Figure 14: Younger Mönkemöller Glass Planimeter sold on eBay (2nd August 2021) by Galerie alte Technik (http://www.galerie-alte-technik.de/).

11. Mechanised planimeter byAnton Ludvig Köhler (18??)

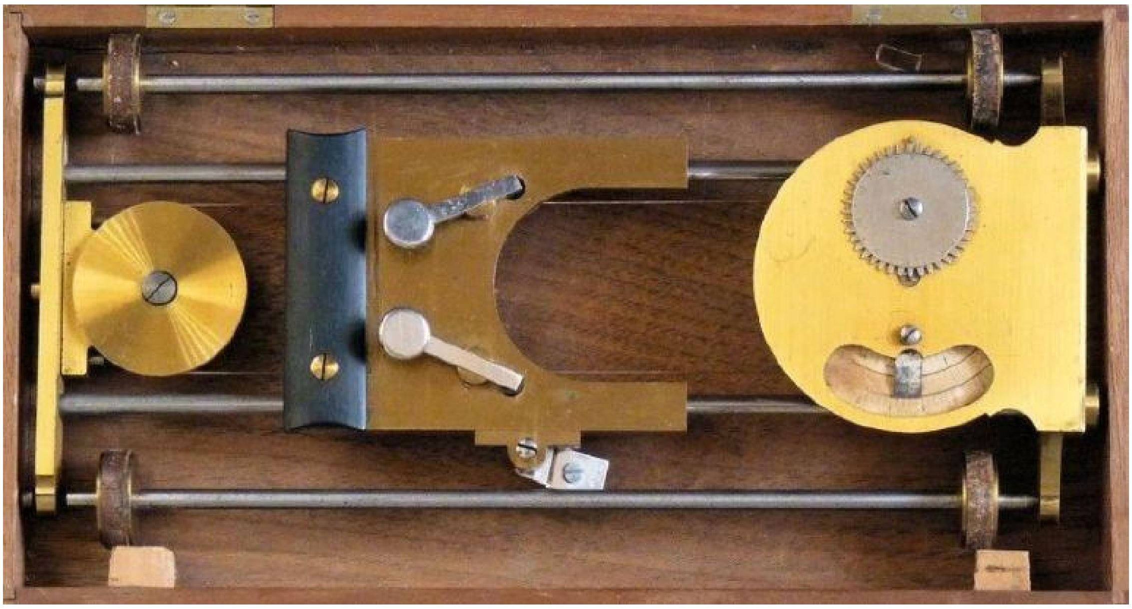


Figure 15: Köhler planimeter top view (Source: Collection Zerfowski).

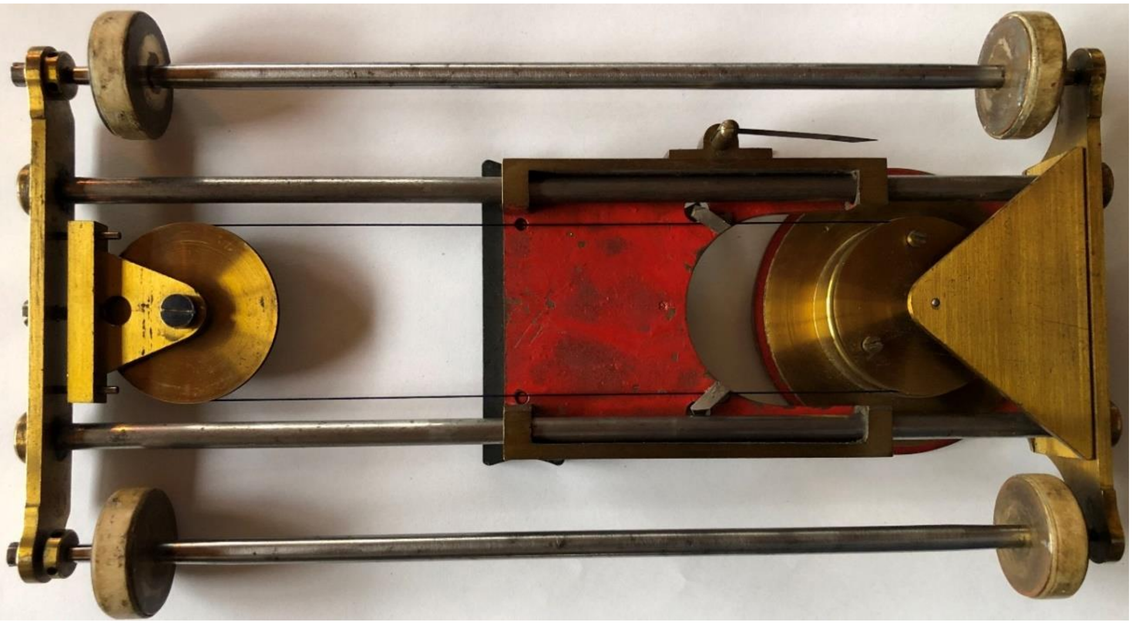


Figure 16: Köhler planimeter bottom view (Source: Collection Zerfowski).

There have been other mechanical realizations than Mönkemöller's approach. In Figure 15 to Figure 17 a completely different design possibly by Anton Ludvig Köhler[11] (*11.10.1828, †06.06.1907, of Wisby, Sweden, [Hop2021]) can be seen. Peter Hopp described further variants of this kind of device in [Hop2021]. Furthermore, two additional copies can be found at the Europeana collection [Eur2020-1, Euro 2020-2]. Due to the similarity of the devices the author assumes the above shown device has also been made by Köhler.

[11]Anton Ludvig Köhler was a Swedish watchmaker, photographer and instrument maker.

Again a glass plate with engraved squares is used. The glass plate is put on the area to be determined. The measuring device is put above the glass plate in such a manner, that the wheels are guided by the edges of the glass plate, such that the device can move horizontally over the glass plate and the area of interest (Figure 17). Now the planimeter can be moved vertically up and down rolling on the wheels. If one knob (not both) on the sled is pressed the visible thread is pinched below the sled (see Figure 16) and moves together with the sled. The moving thread is running around the measurement wheel and turns the wheel accordingly. If the sled knobs are released the thread is not moving and consequently the measuring wheel is not turning.

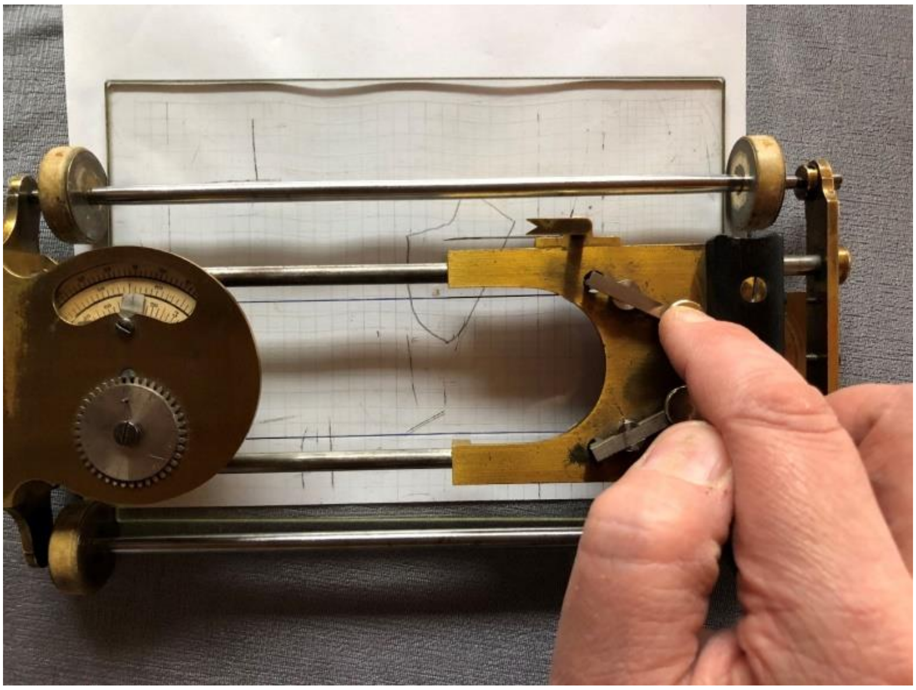


Figure 17: Operating the Köhler planimeter (Source: Collection Zerfowski)

Now we are looking at how the measurement is done.

1. Initially the measurement wheel is manually set to zero. Alternatively one can note down the last value, which would need to be subtracted from the final result shown after the measurement.
2. Do not press any knob on the sled.
3. For each column of squares on the glass plate intersecting with the area of interest:

a) Move the sled up to the intersection of the area boundary with the square column. For aiming the intersection use the end of the sharp indicator on the right side of the sled. For correct measurements one needs to aim in such a manner that the sharp indicator is aligned with triangle shaped targeting help.

b) Now the column measurement starts: Press one knob (not both) and move the sled upward until the sharp indicator reaches the upper boundary in that column. Only then release the knob. During this operation the fixed thread turns the measurement wheel and adds accordingly the inner distance between the boundaries of the area in that column.

4. If there are more square columns intersecting with the area under investigation, go to step 3, and repeat.
5. After all relevant columns have been measured; the measurement wheel shows the final result for the area. (Note: In case one did not reset the measuring wheel in the first step, the initial value on the measurement wheel needs to be subtracted).
6. Finally multiply the result from step 5 with the width of the squares on the glass plate, giving the area of the figure on the paper.
7. In case the figure on the paper has been given with a specific scale (e.g. if the area is part of a map), the result from step 6 need to be multiplied by the scale to obtain the area in real terms.

Unfortunately, no additionally information on the usage of these devices or the year of invention is known. Since no traces of the Köhler's planimeter can be found in the available literature, the author of this paper assumes that this planimeter might not have been widely used. Since several examples of these devices are known from the northern part of Europe, these planimeters might have been used mainly in the Scandinavian region. Of course, this assumption still needs to be confirmed or refuted.

12. Planimeter by Zobel and Müller (1815)

There have been even more complicated devices. In 1815 Johann Georg Zobel, a royal Bavarian trigonometer, together with the mechanical engineer Joseph Müller invented and realised a machine for measuring areas based on harp or thread planimeter principles. A mechanical stepping mechanism moves the measuring device stepwise forward. The addition of strip length is performed with a measuring wheel. A detailed description of the machine has been given by the inventor in a 41 page book [MulZob1815].

The economic benefit of the device (today we would call it the "Return on Investment") is given by Zobel and Müller as well. As a reference the inventors mentioned an average cost for the calculation for areas of one Bavarian square mile of 300 Gulden. For entire Bavaria this would sum up to 480,000 Gulden. At that time 4/5 of Bavaria's area still had to be measured. Therefore, the inventors claimed a cost saving for the Bavarian government of around 200,000 Gulden. As a consequence (probably also due to good advertising) the Bavarian government ordered several machines. Unfortunately Zobel and Müller did not mention how many machines or the price of the machines. However, the device has not been very successful in the market. According to Fischer [Fis2014] at least three machines have been sold to the Bavarian tax authorities. One of these machines is located at the Deutsche Museum Munich. For a detailed description of the machine, including high quality photos of the device we again refer to [Fis2014].

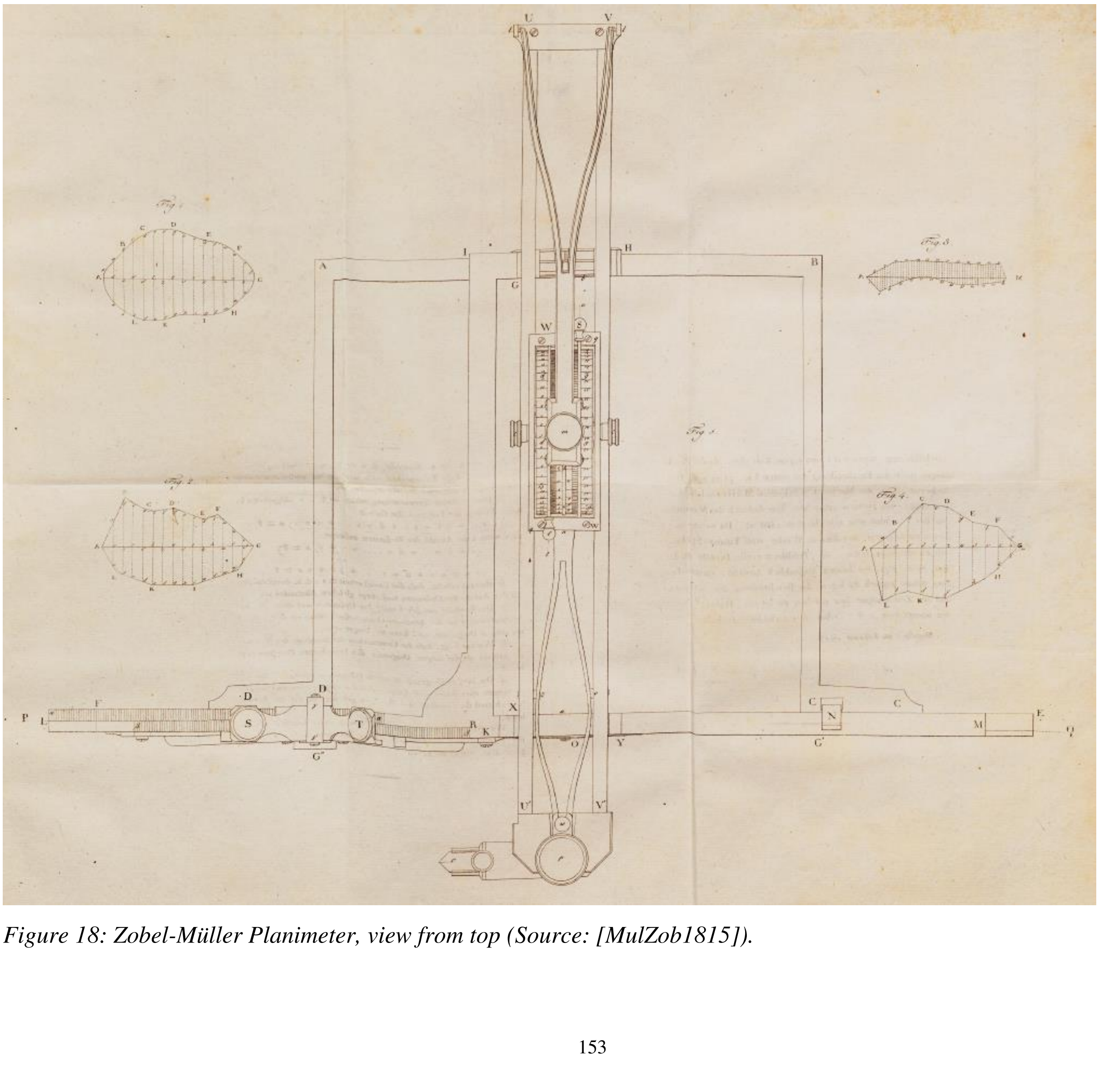


Figure 18: Zobel-Müller Planimeter, view from top (Source: [MulZob1815]).

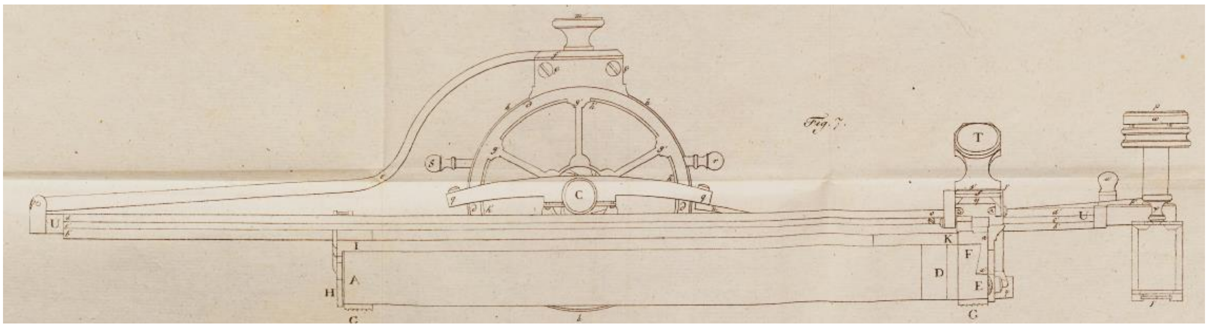


Figure 19: Zobel-Müller Planimeter, side view (Source: [MulZob1815]).

13. Mechanised Hair Planimeter by Beuvière (1844)

Also in France a so called mechanised hair planimeter had been invented. Nearly 30 years after Zobel and Müller's invention, Beuvière designed his devices based on measuring strip length with wheels running inside the figures to be measured. On the internet page [NN20??] photos of two of Beuvière's machines are shown (planimeter no.6 and no. 17). A detailed description of these devices can be found in [Mor1846]. Even though, Beuvière's machines are looking beautiful (and expensive), there is no evidence of a wide spread usage.

14. Planimeter by Oldenburg (1814/1815, post hum published 1825 by Pieper)

Jumping again back some 30 years, at the same time that Zobel and Müller introduced their planimeter solution another device was designed. 10 years after the original invention by Oldenburg in 1815 (be aware that the names Oldenburg and Oldendorp are quite similar and should not be mixed up) his former student Hermann Pieper received permission to publish a detailed description of the Oldenburg Planimeter [Pie1825]. Similar to the Zobel-Möller's device, Oldenburg's machine supports a mechanical stepping for the strips to be measured.

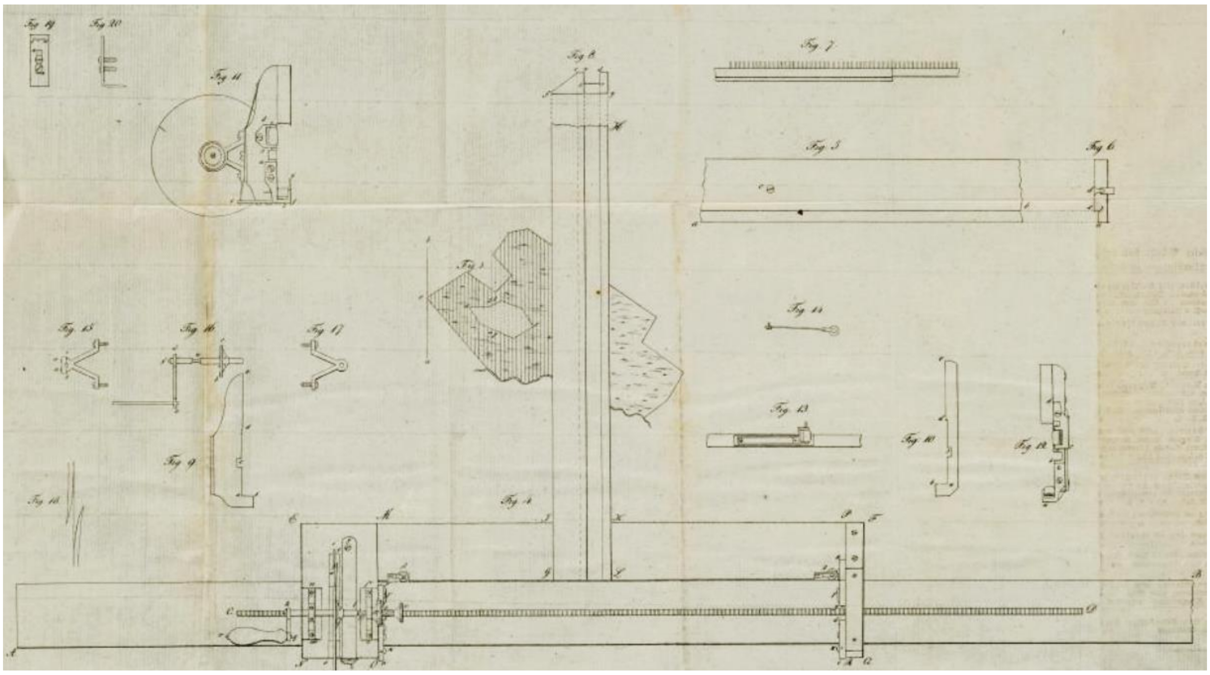


Figure 20: Oldenburg's planimeter: Stepping device for equidistant strip measurements (Source: Pie1825).

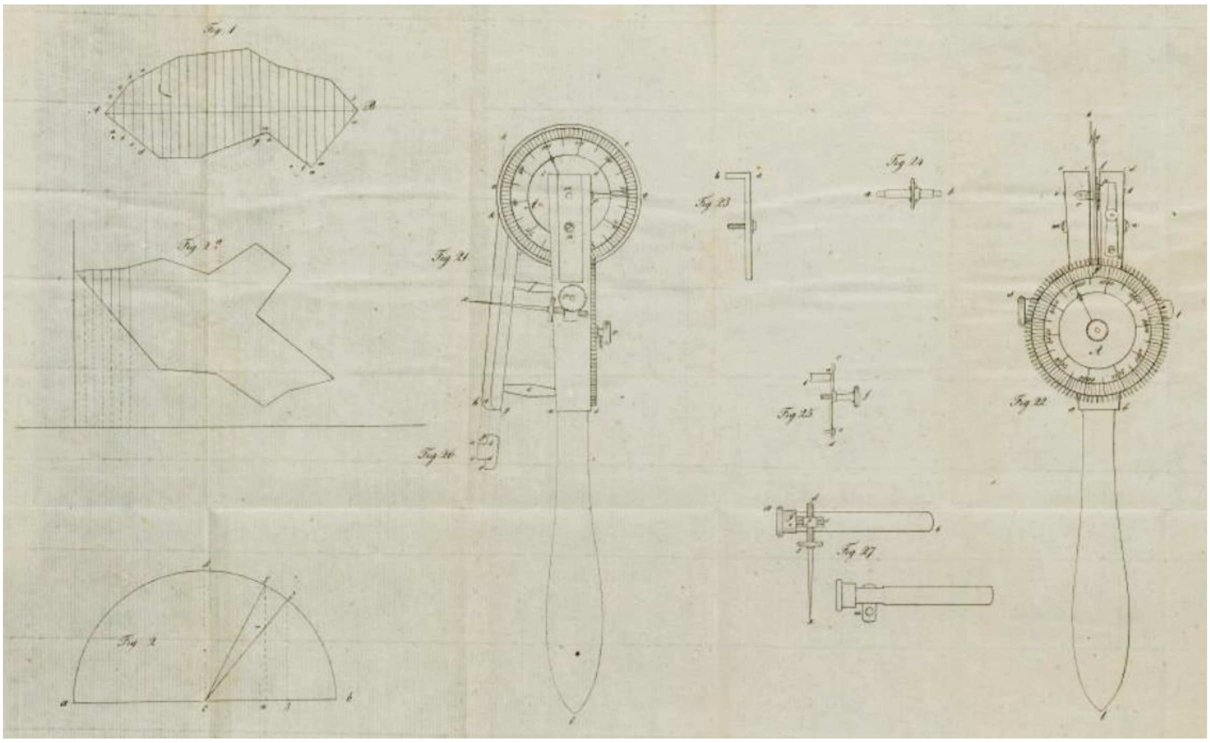


Figure 21: Oldenburg's planimeter: Measuring device, front view on the right, back view in the centre (Source: Pie1825).

One obvious difference of Oldenburg's machine is that it consists of two separate parts: the mechanical stepping device (see Figure 20) and a separate measuring device (see Figure 21).

Regarding the market success of Oldenburg's planimeter, some evidence (at least for the local region of the city Celle in Lower Saxony, Germany) is given in [Pie1825] as well. The device had been manufactured on order and adapted to the individually required scale. Since the main parts of the device had been made out of wood, two different price categories had been offered: 37 Konventionsthaler for devices made of ebony, 31 Konventionsthaler for other wood. According to [Bun2021] 37 Konventionsthaler in 1825 correspond to approximately 1570 Euro in 2020.

Orders had to be sent to the clock maker and engraver Wicke at Lüneburg, who already sold 9 copies to the "Königliche Landes Oeconomie-Collegio" at Celle, a kind of a Royal state tax office.

15. Summary

While reading the article, the reader might have perceived that the history of area measurement in the 19th century has being extremely volatile. The available literature with many detailed discussions on the advantages and disadvantages of the different devices shows the urgent need for these kinds of tools. As a consequence of this demand many inventors and instrument makers tried to enter the market. Many failed but some have been more or less successful.

Last but not least, one short outcome of this article should be mentioned as well. Due to the available information, the author mainly focused on planimeters in the German speaking regions. The author would be glad to receive any additional information on parallel activities on the same subject in other regions of the world.

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