

A dramatic sunset or sunrise over the ocean. The sky is filled with dark, heavy clouds, with a bright sun breaking through on the left side, casting a golden glow. The ocean is dark with white-capped waves in the foreground. In the distance, a pier or breakwater extends into the water on the right side.

Calculating Waves - not only on the Beach

**Mechanical Calculation of
Harmonic Waves by
Harmonic Analysers**

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Mechanical Calculation of Harmonic Waves by Harmonic Analysers

"Théorie analytique de la chaleur" (1872)
(engl. "The Analytic Theory of Heat")

"Any periodic, continuous function can be generated by a series of sine and cosine functions."

Mathematical equation: *Fourier Series*

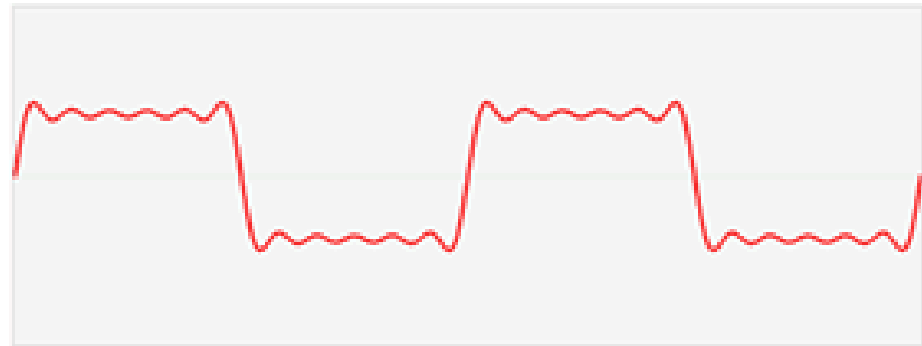
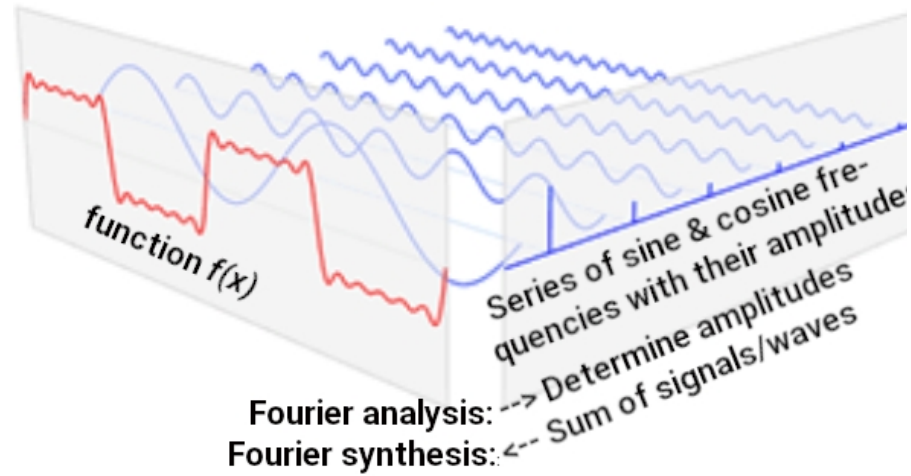
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^N (a_n \cos(n \cdot w \cdot x) + b_n \sin(n \cdot w \cdot x))$$

The values a_0, \dots, a_n and b_1, \dots, b_n are called
Fourier Coefficients



Jean Baptiste Joseph Fourier
Mathematician & Physicist
*21st March 1768
†16th May 1830

Visualisation of the Fourier Series



Mechanical Calculation of Harmonic Waves by Harmonic Analysers

Tidal machine (a harmonic synthesizer)

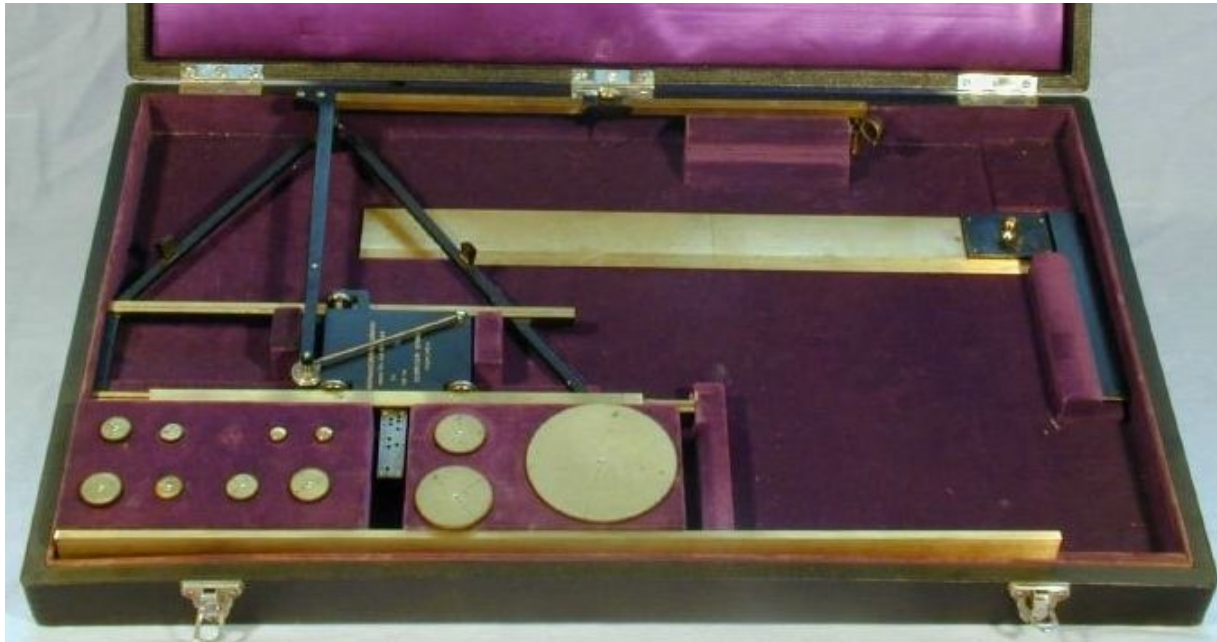
- by William Thomson (Lord Kelvin)
- built 1872/1873
- Based on measured tide heights, harmonic components (periodic rotation of the moon and earth) had been derived.
- Used for predicting the height of tides for different places for up to 2 years
- Location: Science Museum London.
- Obviously it's quite expensive and no desktop device



Mechanical Calculation of Harmonic Waves by Harmonic Analysers

Mader Analysator (a "cheap" desktop device)

Invented by 1909 Produced by Stärzl till ca. 1930
Latest known serial number: 190



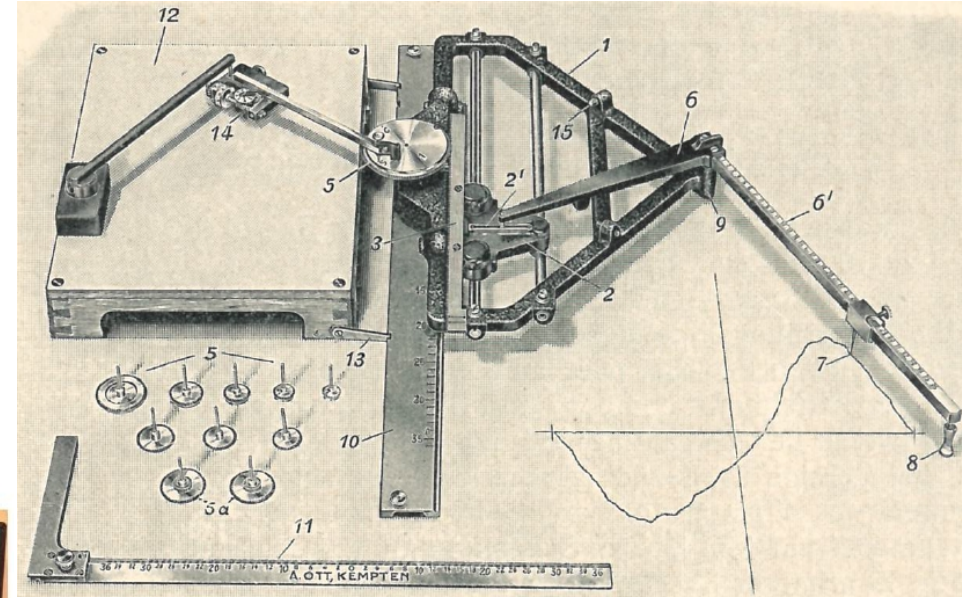
Typical device plate of Mader-Stärzl analysers.

Mechanical Calculation of Harmonic Waves by Harmonic Analysers

Mader-Ott Analyser

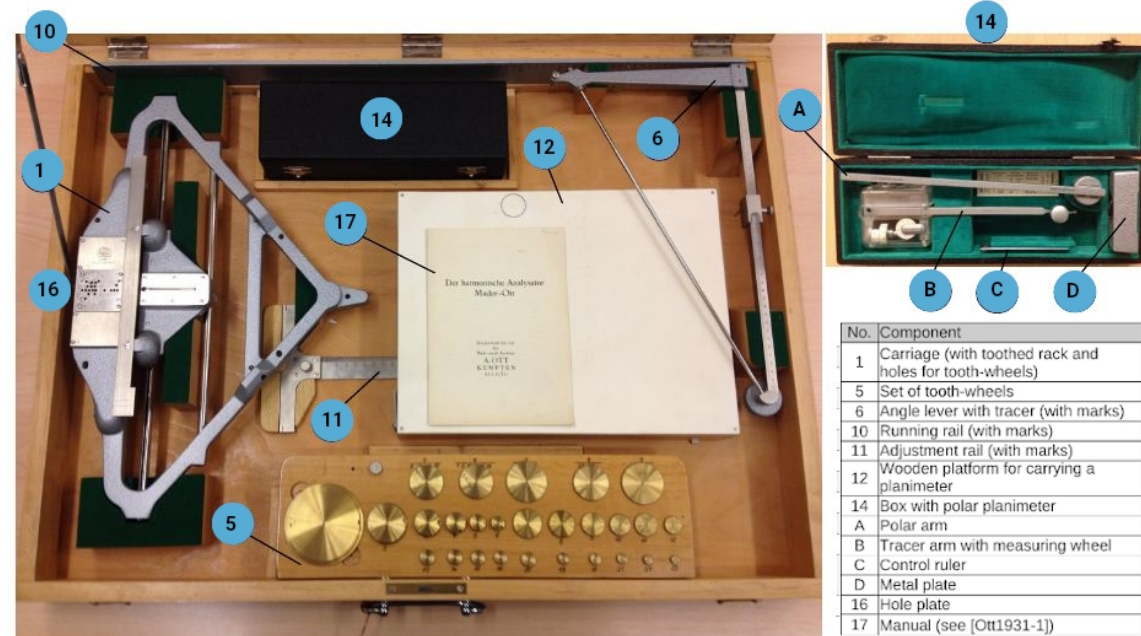
(1929 start of production by Ott without agreement with Mader)

- Storage box with (bottom)
- Set-up of operating analyser (right)



Operation

- The tracer (7) is guided over the curve
- The small carriage (2) moves vertically within bigger carriage (1)
- The carriages move a toothed bar (3), which moves a gear (5),
- which moves a planimeter (14)
- The differences of the planimeter readings gives the Fourier coefficient



No.	Component
1	Carriage (with toothed rack and holes for tooth-wheels)
5	Set of tooth-wheels
6	Angle lever with tracer (with marks)
10	Running rail (with marks)
11	Adjustment rail (with marks)
12	Wooden platform for carrying a planimeter
14	Box with polar planimeter
A	Polar arm
B	Tracer arm with measuring wheel
C	Control ruler
D	Metal plate
16	Hole plate
17	Manual (see [Ott1931-1])

Mechanical Calculation of Harmonic Waves by Harmonic Analysers

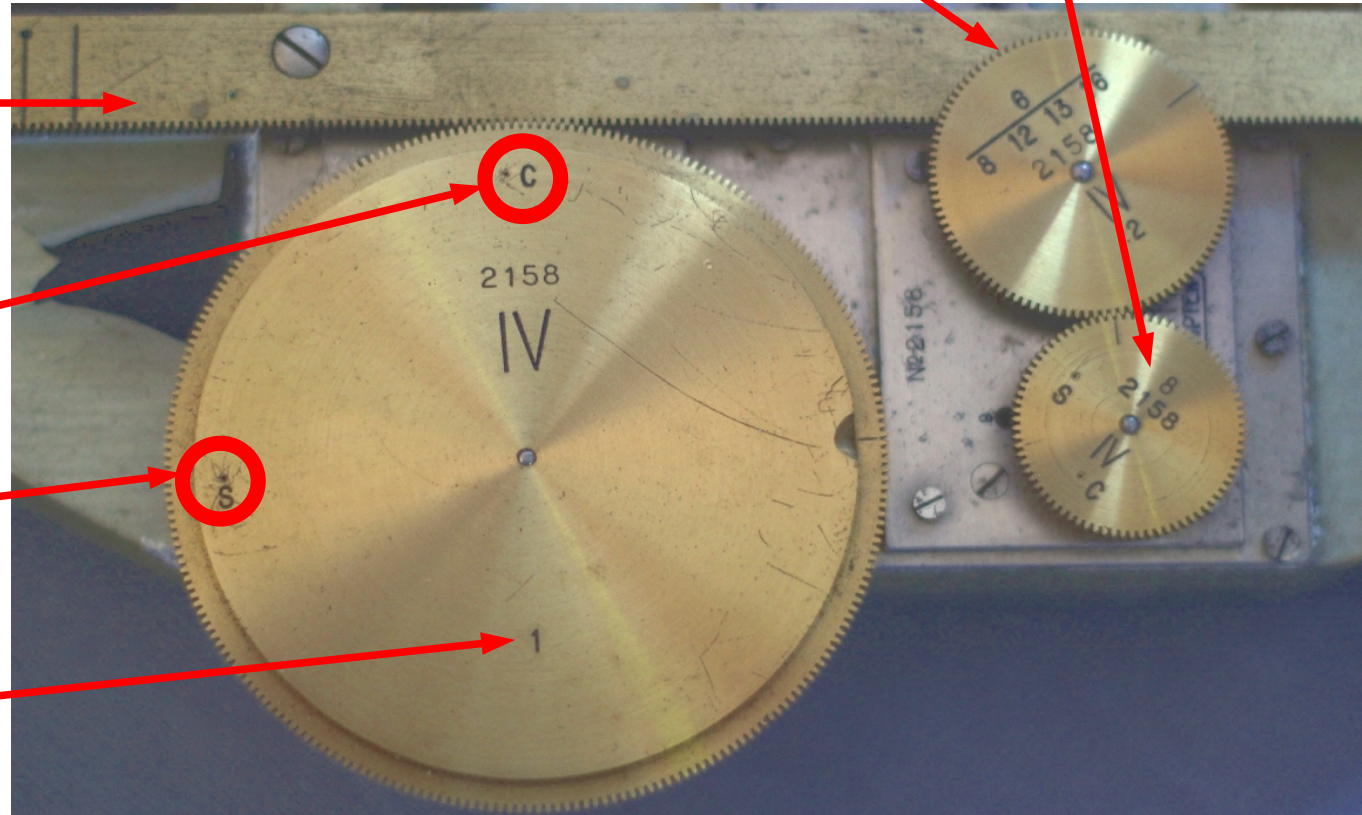
Operating gears

Toothed bar moved
by carriage

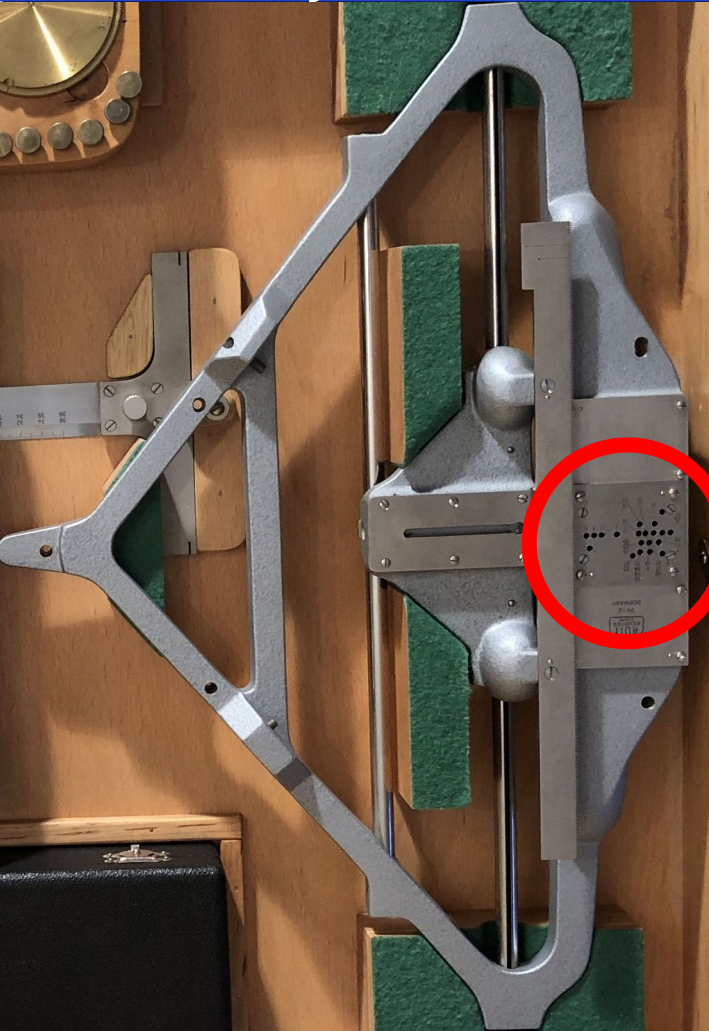
Joining holes for planimeters:

- C for measuring cosine coefficient
- S for measuring sine coefficient
- Gear for coefficient a_1 and b_1

Intermediate gear
Gear for coefficients a_8 and b_8



Mechanical Calculation of Harmonic Waves by Harmonic Analysers

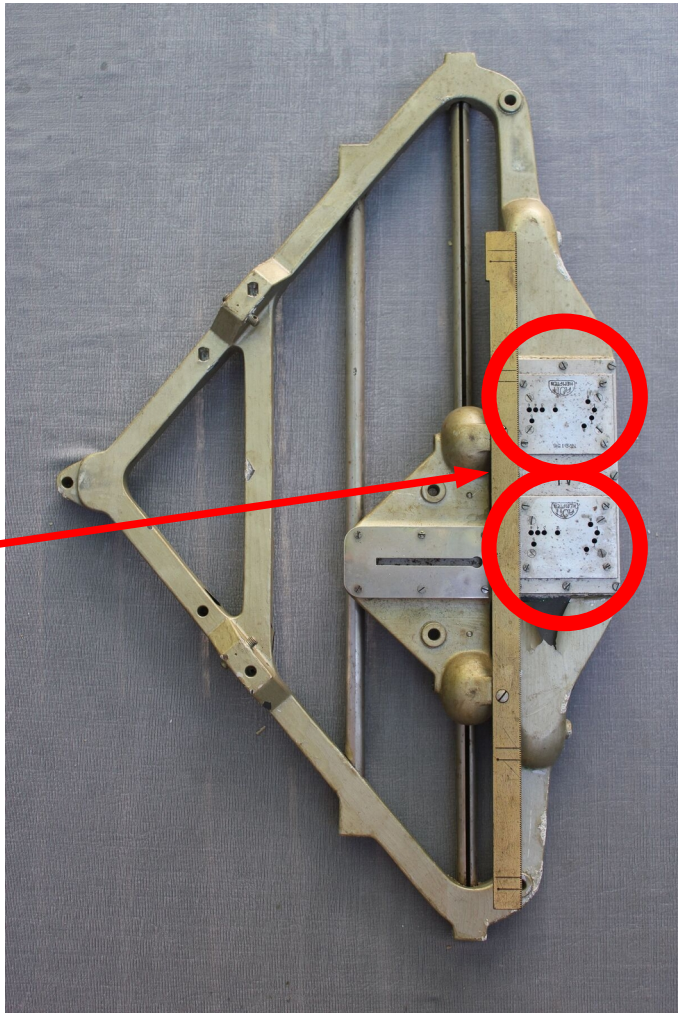


Analysers Variance: Hole plates

Carriages with
one hole plate

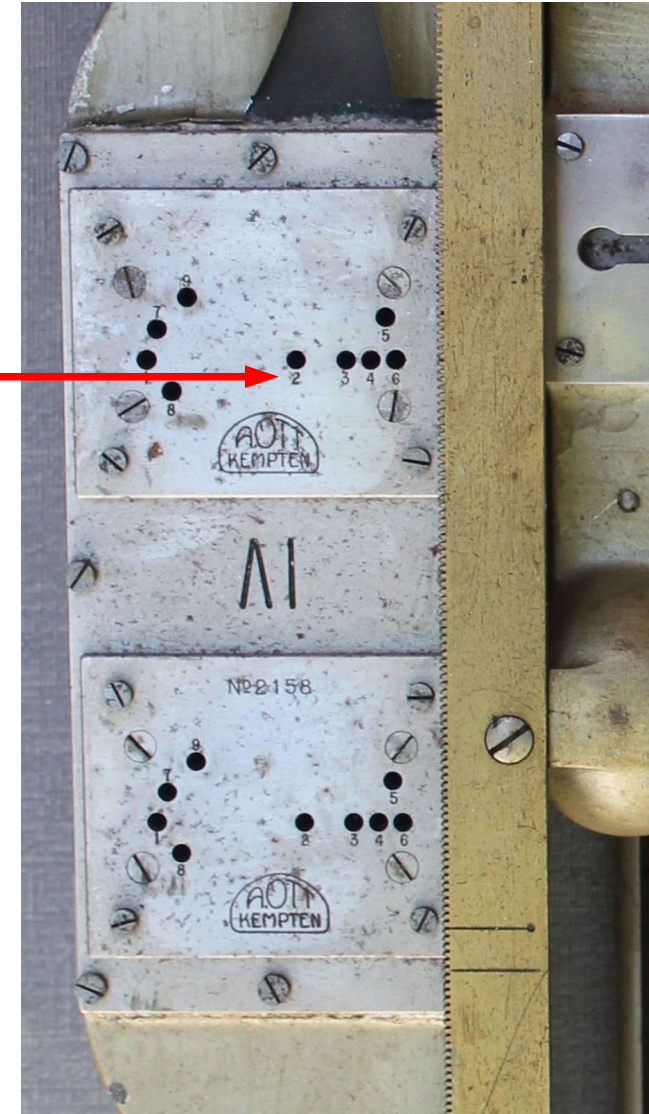
&

double hole plates
(double analyser)



Analysers Variances: Hole plates Double analysers

- Each hole is labeled with a number →
- Indicating which gear to be used in which hole
- Double hole plates introduced by 1941
- Allows simultaneous operation of two gears and therefore two planimeters
⇒ determination of two Fourier coefficients while tracing function $f(x)$ once



Analyser Variance: Gear sets (2 x 48 gears)



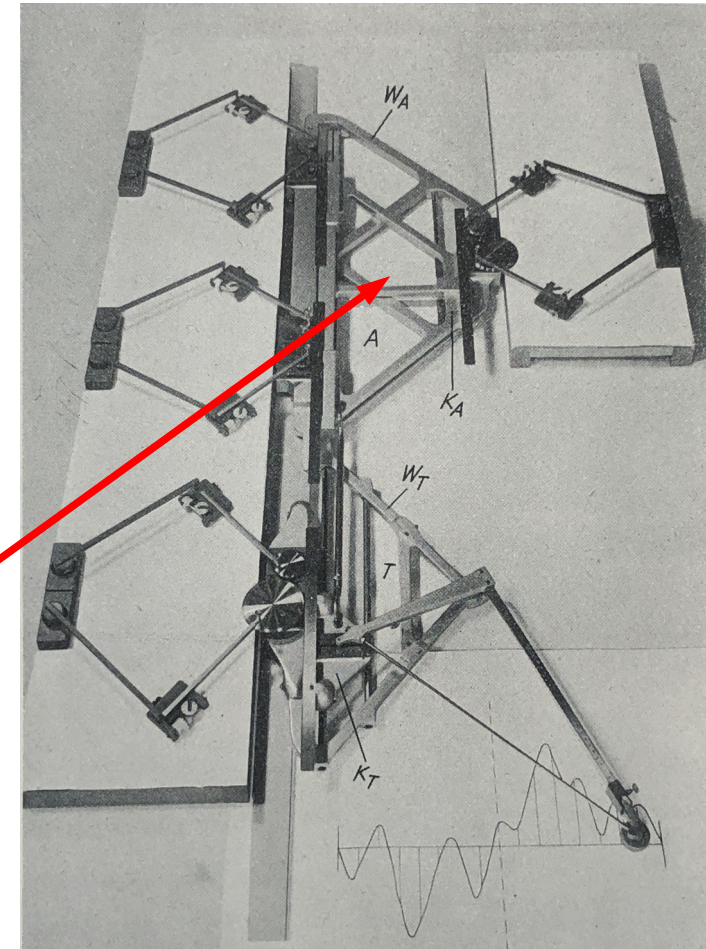
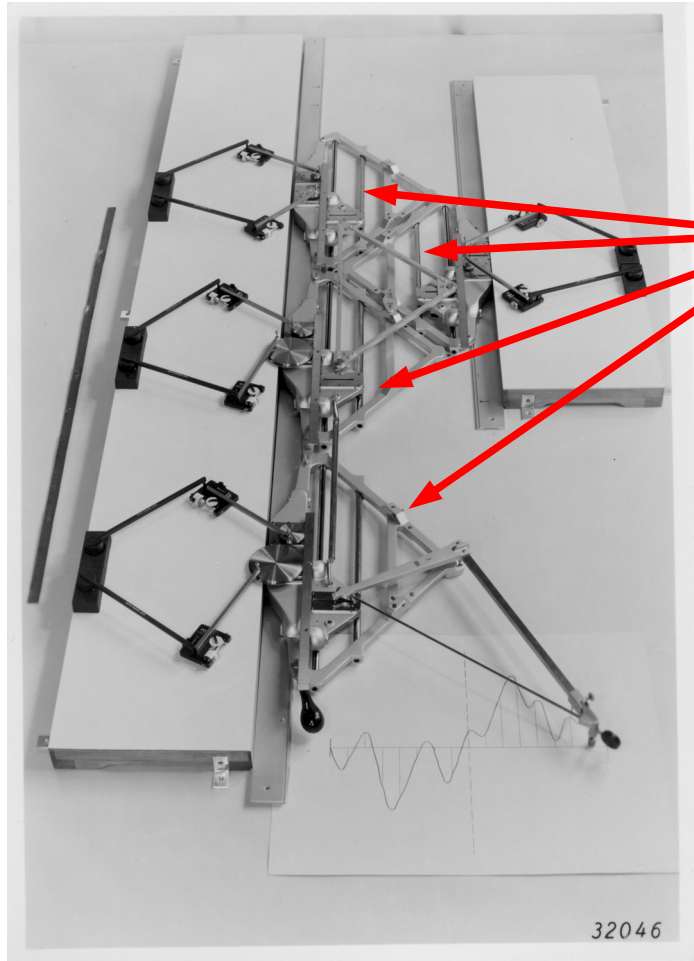
for determining
sine and cosine
coefficients of
higher
frequencies
⇒ leading to
higher
accuracy

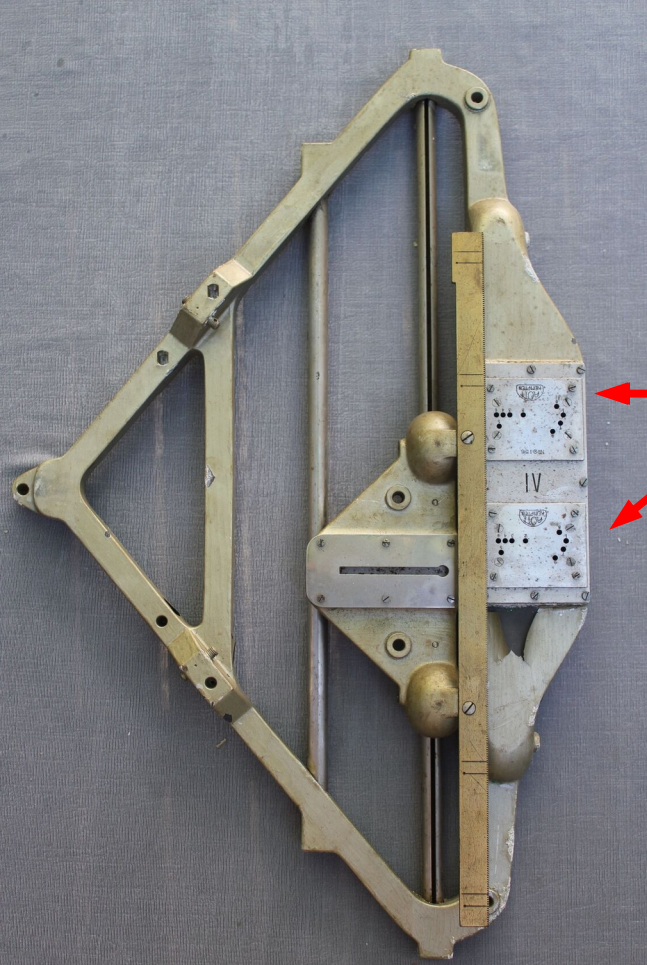
Analysers Variance: Carriage trains

Train with 4 carriages for
4 x 2 planimeters
(April 1941)

&

Train with 2 carriages for
1 x 2 + 1 x 6 planimeters
Extendable with 2nd carriage
for 1 x 6 planimeters.
In total, a 14-fold analyser.
(1942)



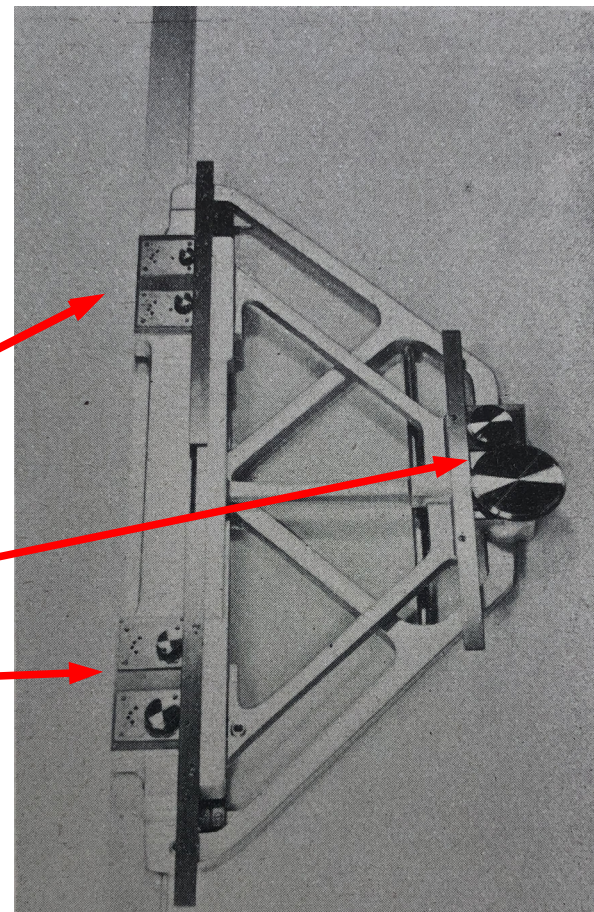


Analysers Variance: Carriage type

Carriages with
double hole plates
(for 2 planimeters)

&

3 x double hole plates
(for 6 planimeters)



Mechanical Calculation of Harmonic Waves by Harmonic Analysers

Detlef Zerfowski | 21.09.2019

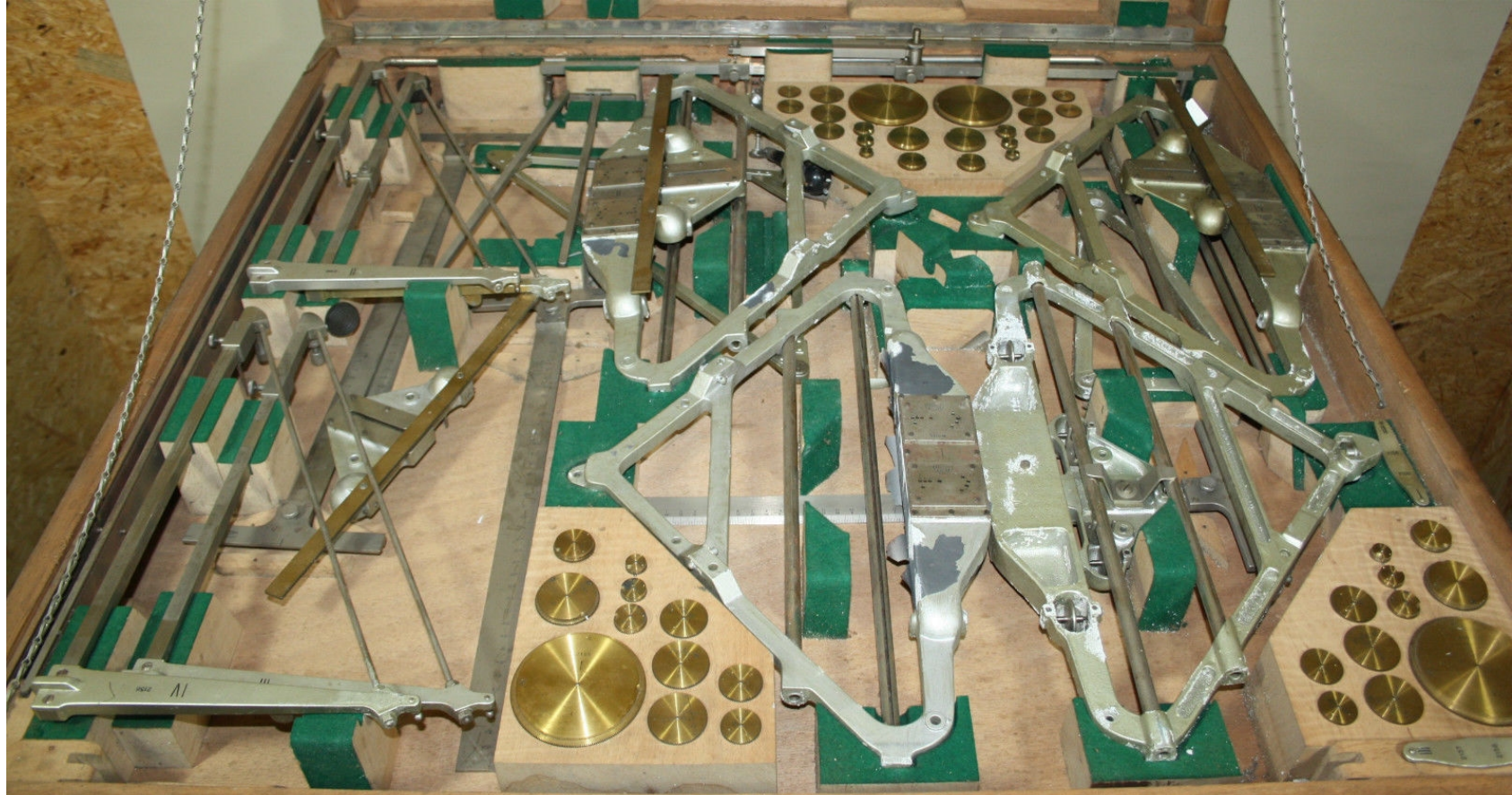
Analysers Variances: 8-fold analyser

Storage box:

Size: 1 sqm
Weight: 45 kg
Year: 1941

with

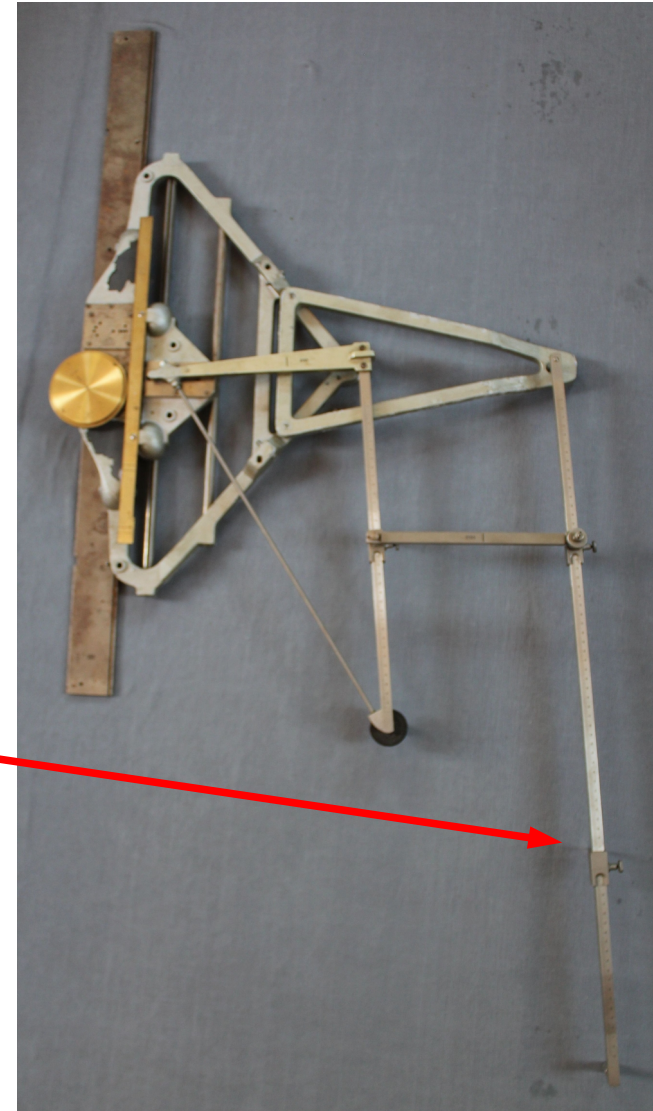
4 carriages
4 gear sets with
11 gears each
1 tracer arm
extension



Analyser Variances: Trace arm extension

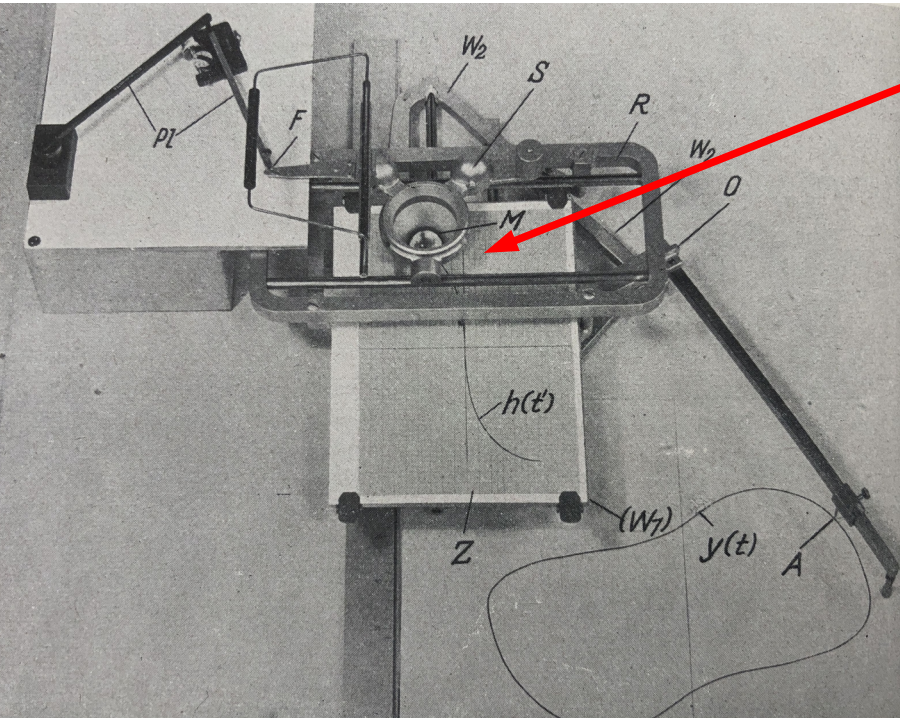
Without extension function curve of
36 cm width x 16 cm height
could be analysed

With the tracer arm extension curves of
72 cm width x 16 cm height
could be analysed



Analyser Variance: Stieltjes Planimeter For determining the Stieltjes Integral

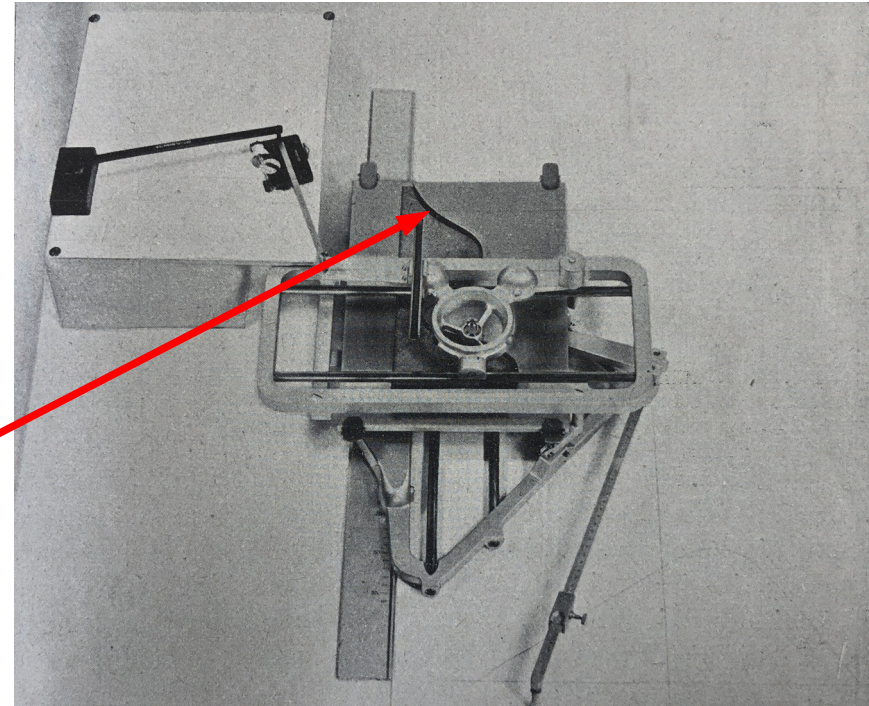
$$\oint y(t) dh(t')$$



manually tracing of $h(t')$

or

enforced tracing of $h(t')$



Late Adopter: Stanley Harmonic Analyser

Manufacturing:
1950th till 1965

Three models:
U8101 (single hole plate)
U8102 (double analyser)
U8103 (with 2 set of gears)

Only 55 to 60 devices built

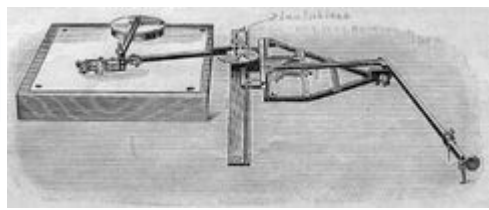


Source: Tekniska Museet:

<https://digitaltmuseum.org/021026361373/harmonisk-analysator>

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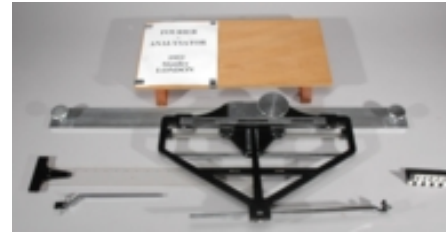
Invention of analyser by Mader



$\sum \approx 200$
Stärzl

Start of production by company Stärzl

End of production by company Stärzl



$\sum \approx 60$
Stanley

Start of Mader type Stanley analysers

End of production by Stanley

1910

1920

1930

1940

1950

1960

1970

Company Ott starts production of Mader-Ott analyser



Introduction of double-analyser and 8-fold analyser



Improved carriage for 6 planimeters



End of production of Mader-Ott analysers

$\sum < 600$
Ott

Number of produced Mader based harmonic analysers

$\sum_{all} < 900$

Questions & Answers

Analog: Locally @ the plenum

Digital: Detlef@Zerfowski.com

